$$\nu A \to \nu A^{(*)}$$

Coherent neutrino-nucleus scattering

Theory and Experiment

Dmitry V. Naumov
JINR

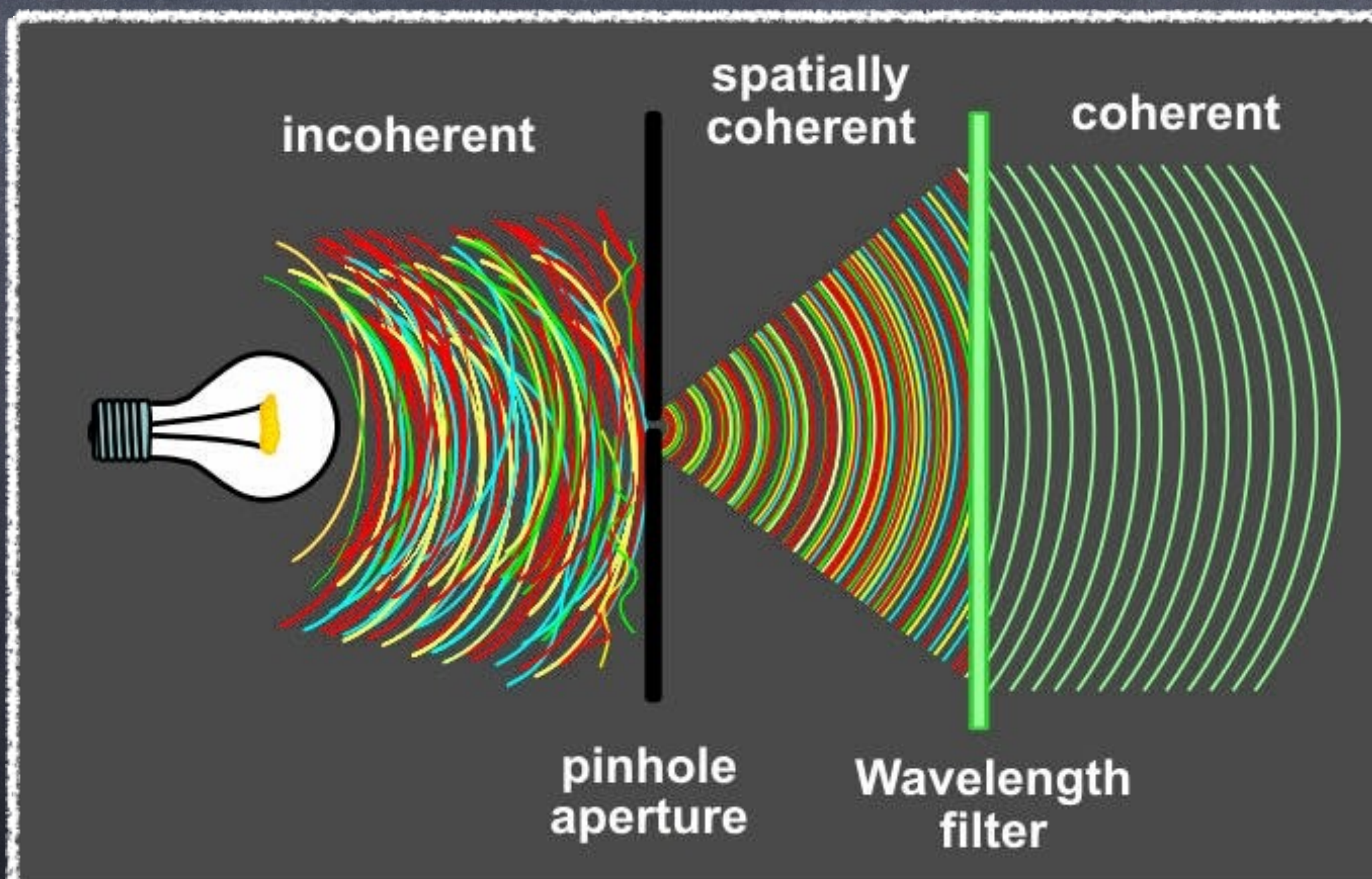
Definitions

- 3-vectors by bold face
- Dirac gamma-matrices γ^μ
- $\hbar = c = 1$

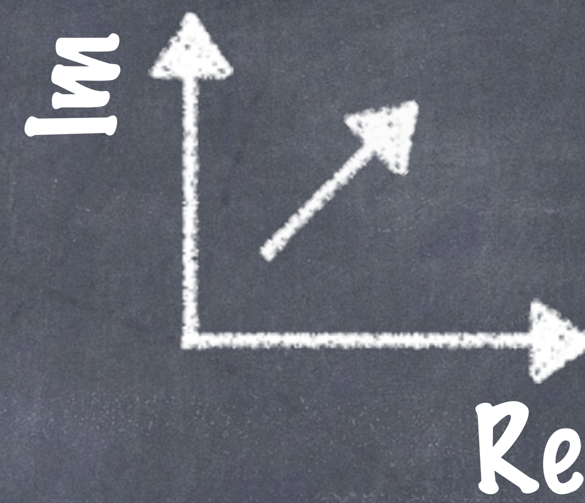
Relative phase of two waves is

Constant → Coherent

Random → Incoherent



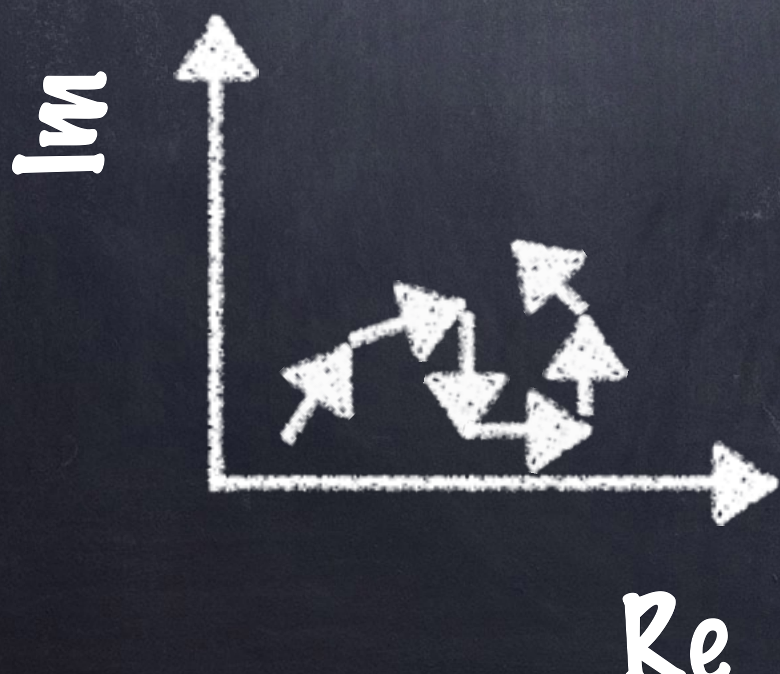
- Let us present a wave amplitude by a vector in the complex plane



- Coherent sum of N amplitudes

$$\propto N \longrightarrow |\sum_k A_k|^2 \propto N^2$$

- Incoherent sum of N amplitudes



$$\propto \sqrt{N} \longrightarrow |\sum_k A_k|^2 \propto N$$

Cross-section
in particle physics

- Number of interactions in one second

$$\frac{dN_{int}}{dt} = \Phi_A N_B \sigma_{AB}$$

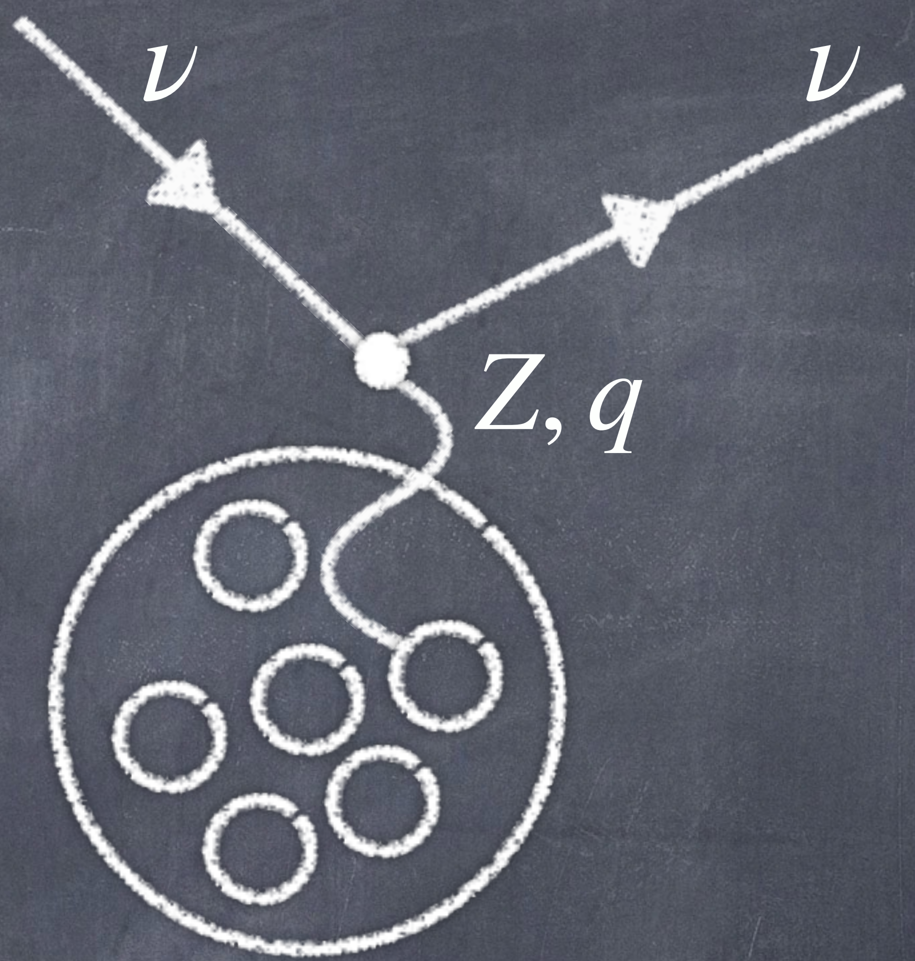
- The cross-section

$$\sigma_{AB} \propto |\mathscr{A}|^2$$

Neutrino-Nucleus Coherent Scattering

Introduction to Freedman's Theory

D.Z. Freedman. Phys.Rev. D9 1389 (1974)



- Assuming nucleons have definite 3-coordinates $\mathbf{x}_k$ the amplitude

$$\mathscr{A} = \sum_{k=1}^{N} \mathscr{A}_k e^{i\mathbf{q}\mathbf{x}_k}$$

- If $\mathbf{q}\mathbf{x}_\mathbf{k} \approx \mathbf{q}\mathbf{x}_\mathbf{j}$ for any k, j amplitudes sum up coherently

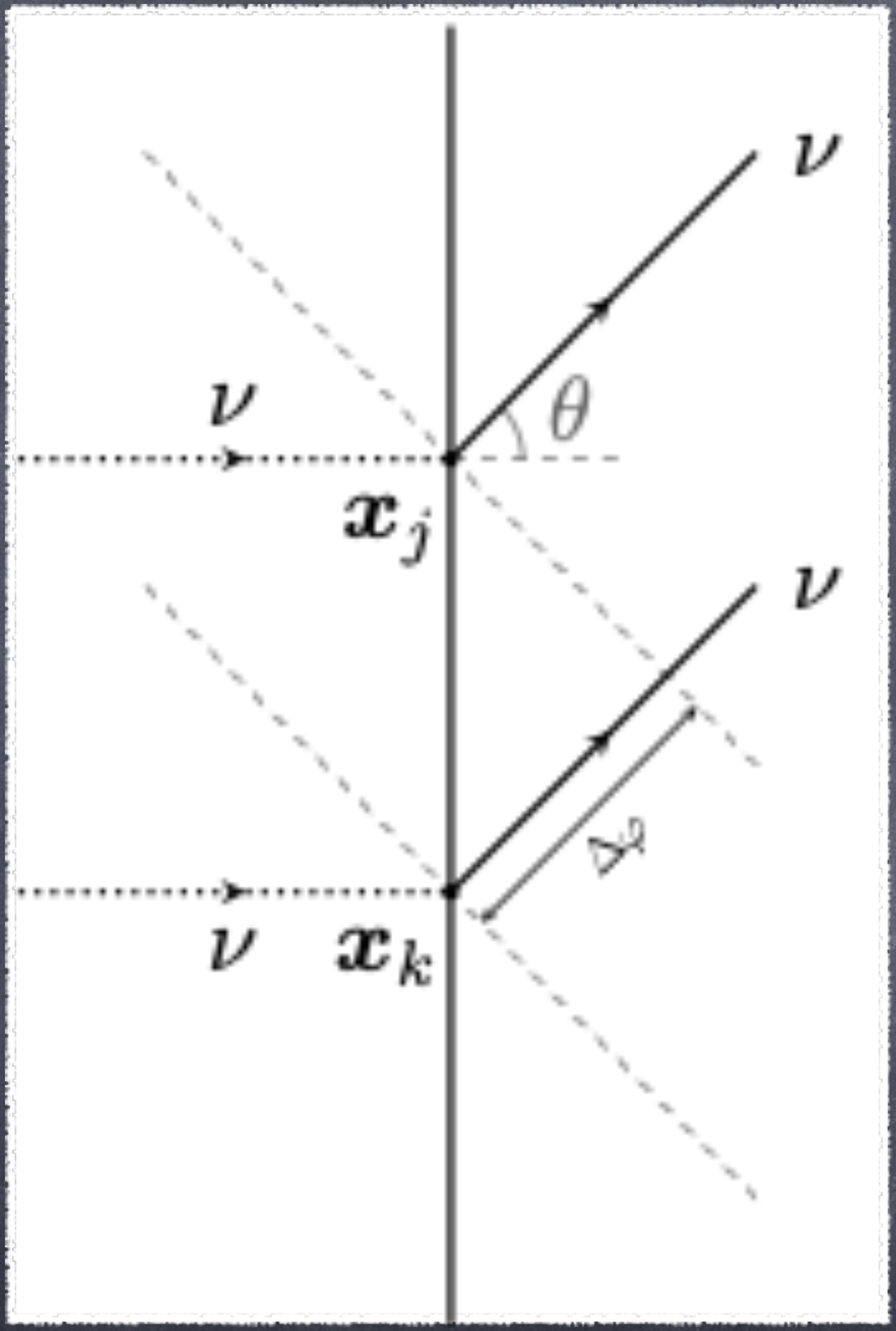
$$\mathscr{A} \simeq NF(\mathbf{q})\mathscr{A}_0$$

Coherence condition
$|\mathbf{q}|\mathbf{R} \ll 1$

- Nucleus Form-factor

$$F(\mathbf{q}) = \int \mathbf{dx}\rho(\mathbf{x})\mathbf{e}^{\mathbf{iqx}}$$

Loss of coherence

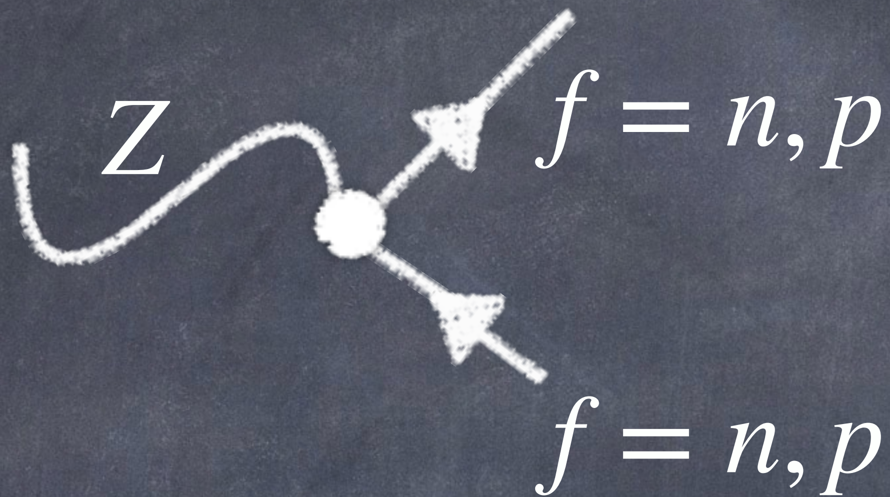


- Difference of phase of scattered waves leads to a loss of coherence $\Delta\varphi = \mathbf{q}(\mathbf{x_j} - \mathbf{x_k})$

Coherence condition
$|\mathbf{q}|R \ll 1$

Coherent cross-section

- The vertex



$$\bullet = \gamma^{\mu}(g_V^f - g_A^f \gamma_5)$$

- The cross-section for spin-even nucleus

$$\sigma \propto \left| g_V^n N F_n(\mathbf{q}) + \mathbf{g_V^p Z F_p}(\mathbf{q}) \right|^2 \simeq N^2 (g_V^n)^2 \,|\, F_n(\mathbf{q}) \,|^2,$$

- Axial couplings do not contribute for spin-even nucleus
- Protons do not matter $\quad g_V^p = \frac{1}{2} - 2\sin^2\theta_W \approx 0.023$

Coherent cross-section

- N^2 times larger than on a free nucleon
- N times larger than an incoherent scattering off a nucleus
- Observable: kinetic energy (T_A) of the scattered nucleus

$$T_A = \frac{\mathbf{q}^2}{2M_A} = 5\text{eV}\left(\frac{q}{\text{MeV}}\right)^2\left(\frac{100\text{GeV}}{M_A}\right)$$

- Very hard to detect so small kinetic energy!
- Go to large q? $q \ll \frac{1}{R_A} \simeq 40\text{MeV}$

$$\hookrightarrow T_A(40\text{MeV}) = 8\text{keV}$$

Coherent and incoherent
Short summary so far

- **Coherent:**
  - N^2 dependence
  - $E_\nu \ll 20\text{MeV}$

Coherence condition
$|\mathbf{q}|\mathbf{R} \ll 1$

$$|\mathscr{A}|^2 = |\sum_k A_k|^2 \simeq N^2 |A_0|^2$$

- **Incoherent:**
  - N dependence
  - Can be obtained assuming N independent scatterers

$$|\mathscr{A}|^2 = \sum_k |A_k|^2 \simeq N |A_0|^2$$

Neutrino-Nucleus Coherent Scattering

Experiment

How to detect some keV kinetic energy of a nucleus?

(i) Scintillator: Ionization —> Photons —> Photoelectrons

- Energy of one photon with $\lambda = 300\text{nm}$ is about 4 eV
- How many such photons could be produced with 1 keV?
- 250 is an upper bound.
- For liquid scintillators 8-12 photons/keV
- For solid state scintillators, like NaI, 40 photons/keV
- These numbers are for gamma or electron with 1 keV
- A nucleus produces an order of magnitude less number of photons

How to detect some keV kinetic energy of a nucleus?

(i) Scintillator: Ionization —> Photons —> Photoelectrons

- In liquid scintillators a nucleus

  1 photon/keV —> 0.2 p.e./keV

- In solid state scintillators, like NaI, a nucleus

  4 photons/keV—> 1p.e./keV

How to detect some keV kinetic energy of a nucleus?

(ii) Semiconductor detector:

Ionization —> electrons and holes —> electric signal

- Energy needed to produce a pair: electron+hole is about 1-2 eV

1 keV electron —> 500-1000 electrons

which give a detectable current

- Ge detectors @nuGEN. The lowest energy of
  - electrons is 200 eV
  - Ge nucleus is about 1 keV

How to detect some keV kinetic energy of a nucleus?

(ii) Liquid Argon TPC:

Ionization —> electrons and photons —> photons @SiPM

- DarkSide. The lowest energy of Ar nucleus is about 0.6 keV

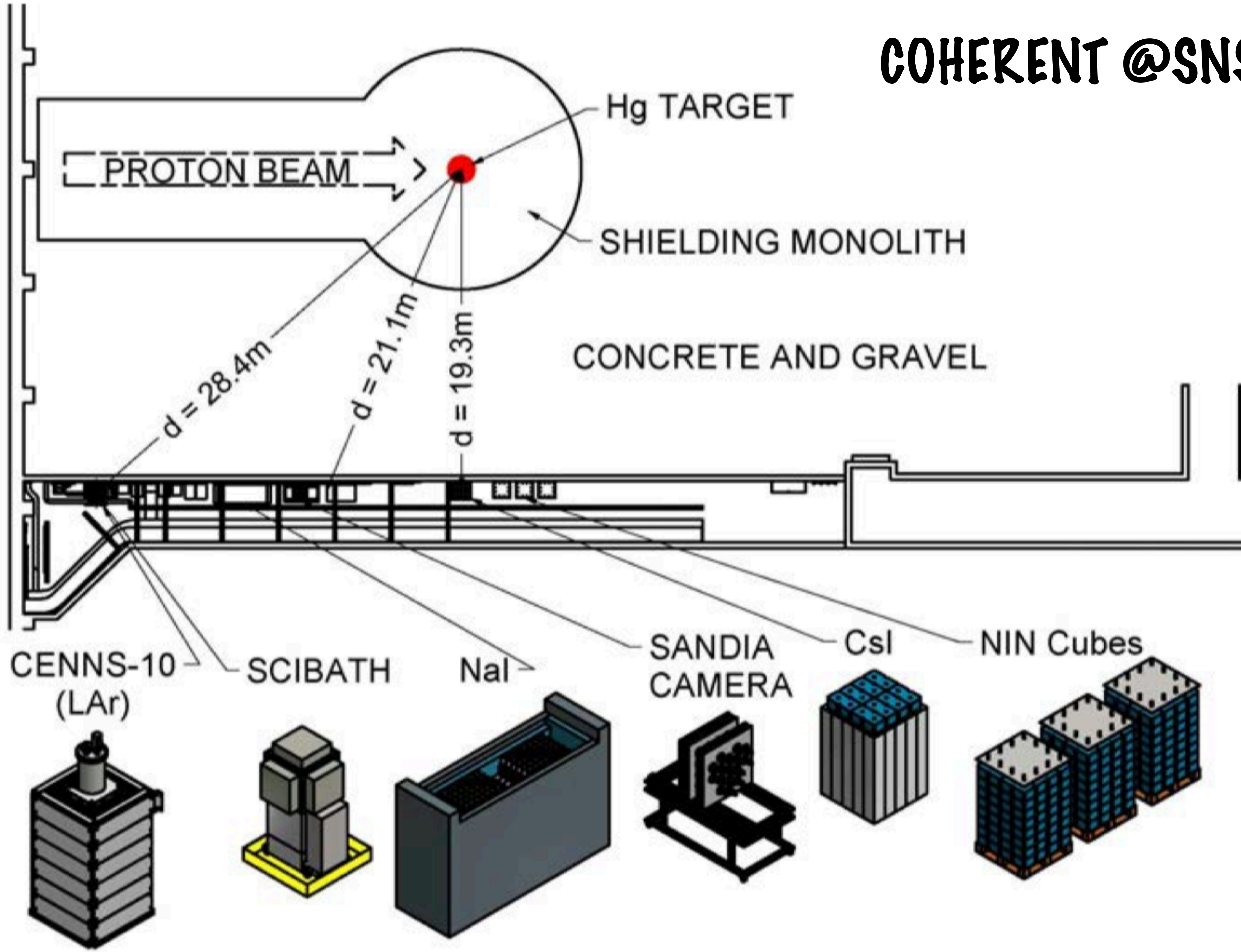
State-of-the-art summary

	COHERENT	nuGEN	DarkSide
Energy Threshold, keV	5	1	0.6
Detector	CsI (NaI)	Ge bolometer	LAr TPC

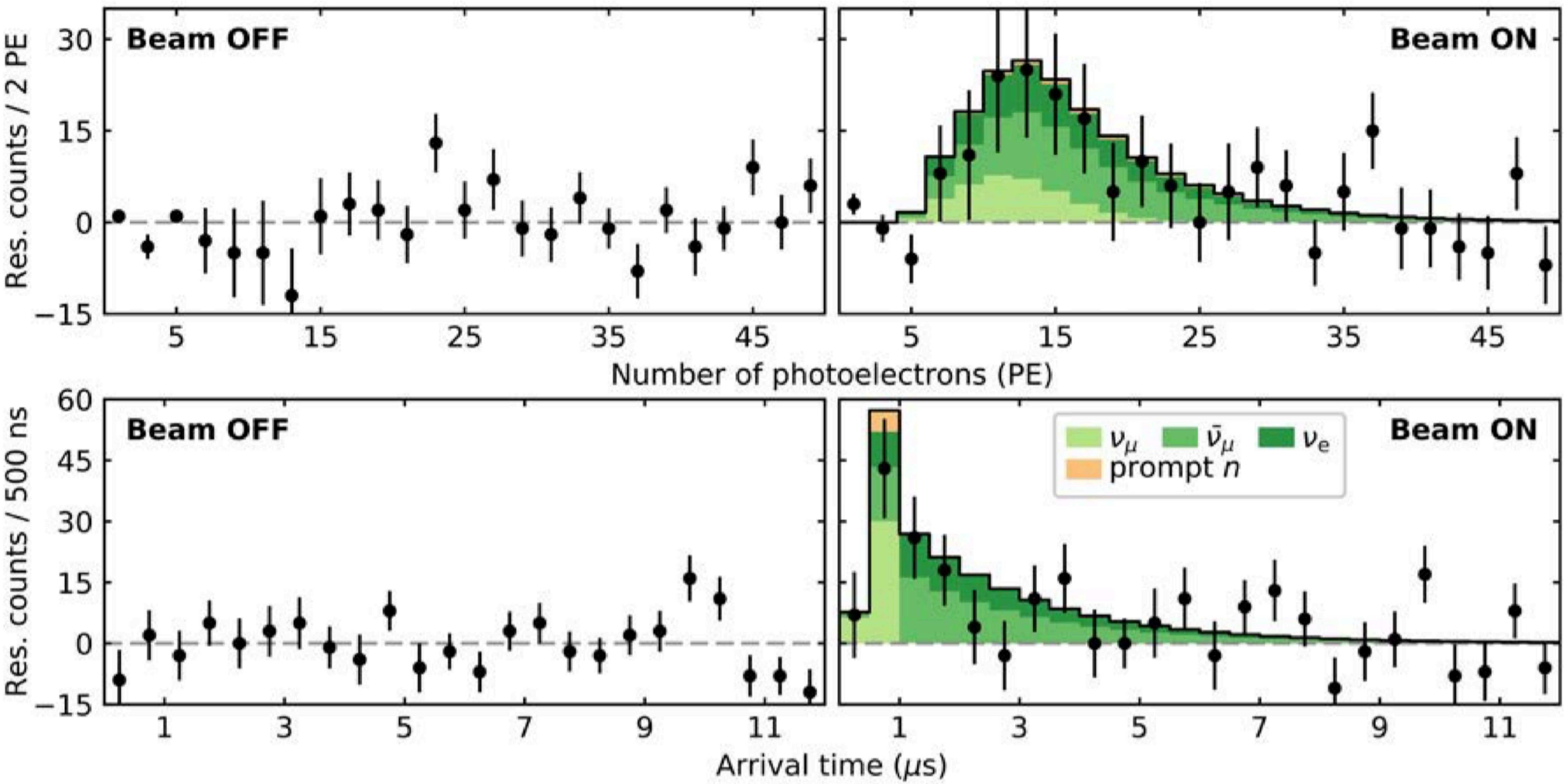
DISCOVERY of COHERENT scattering by COHERENT Collaboration

- Spallation Neutron Source @ Oak Ridge National Laboratory
- 60 Hz 1 μs wide spills
- Strong background suppression due to beam timing
- Neutrino energy is some tens of MeV
- 1.18 p.e./keV and expected T_A is of the order of 15 keV
  - 18 p.e. for the signal. Hard but possible if beam timing is possible

COHERENT @SNS



D. Akimov et al. (COHERENT), Science (2017), 10.1126/sci- ence.aao0990

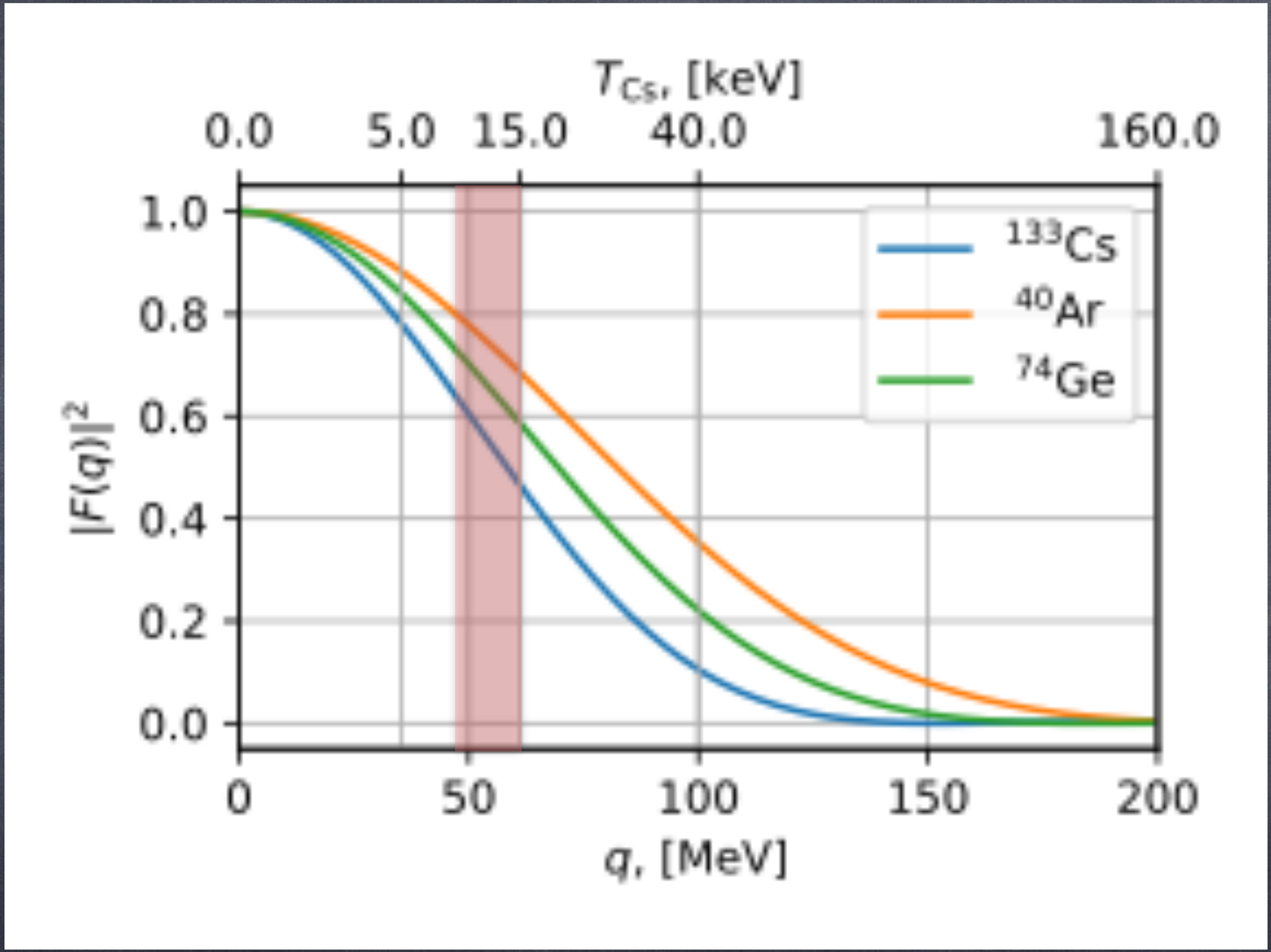


Significance 6.7 σ

Neutrino-Nucleus Coherent Scattering

Back to Theory

- The cross-section $\sigma \propto N^2 |F_n(\mathbf{q})|^2$ vanishes at large q



- Neutrino does not interact with a nucleus at large q?!
- Where is an incoherent scattering? qR=(1,2.7)

- Literature on neutrino-nucleus scattering lacks a formula for both coherent and incoherent scattering
- At the same time for electrons, gamma and neutron scatterings such considerations do exist
- So, our motivation was to build an appropriate theory for neutrino-nucleus scattering from first principles

V.A. Bednyakov, D.V. Naumov. Phys.Rev. D98 (2018) no.5 053004

Revising the paradigm

- Freedman assumed nucleons at fixed positions

$$\mathscr{A} = \sum_{k=1}^{A} \mathscr{A}_k e^{i\mathbf{q}\mathbf{x}_k}$$

- Also, silently he assumed nucleus remains in the same state

- With multi-particle wave-function $\psi_{n/m}(\mathbf{x}_1 \ldots \mathbf{x}_\mathbf{A})$

$$\mathscr{A}_{nn} = \sum_{k=1}^{A} \mathscr{A}^k_{nn} f^k_{nn}(\mathbf{q}),$$

- where $f^k_{mn}(\mathbf{q}) = \left\langle \mathbf{m} \,|\, \mathbf{e}^{\mathbf{iq}\,\widehat{\mathbf{X}}_\mathbf{k}} \,|\, \mathbf{n} \right\rangle$ replaces $e^{i\mathbf{q}\mathbf{x}_\mathbf{k}}$

- Elastic process. Nucleus remains in the same state $|m\rangle = |n\rangle$

$$\lim_{q\to 0} \left\langle n | e^{i\mathbf{q}\widehat{\mathbf{X}}_{\mathbf{k}}} | n \right\rangle = 1 \longrightarrow |\mathscr{A}|^2 = A^2 |\mathscr{A}_0|^2$$

$$\lim_{q\to \infty} \left\langle n | e^{i\mathbf{q}\widehat{\mathbf{X}}_{\mathbf{k}}} | n \right\rangle = 0 \longrightarrow |\mathscr{A}|^2 \to 0$$

These are features reminding 'coherent' scattering

- Inelastic process. Nucleus changes the state $|m\rangle \neq |n\rangle$

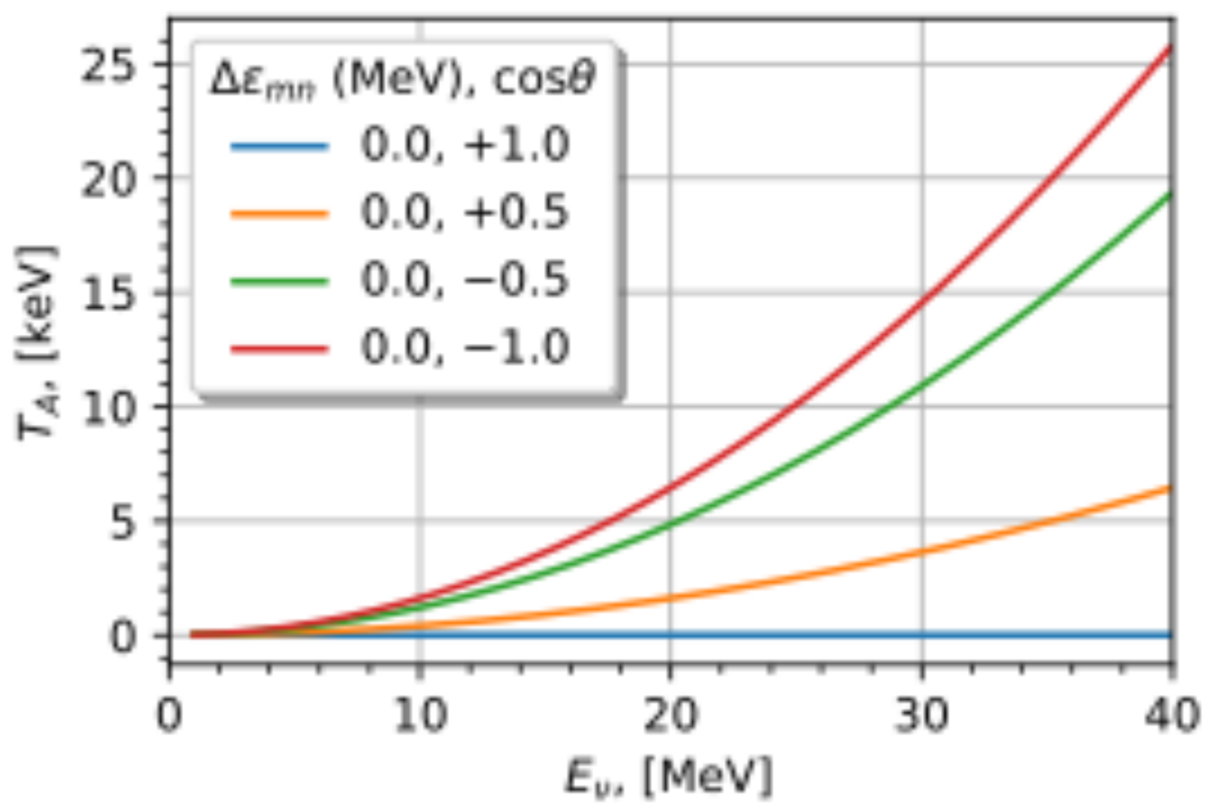
$$\lim_{q\to 0} \left\langle m | e^{i\mathbf{q}\widehat{\mathbf{X}}_{\mathbf{k}}} | n \right\rangle = \delta_{nm} = 0 \longrightarrow |\mathscr{A}|^2 \to 0$$

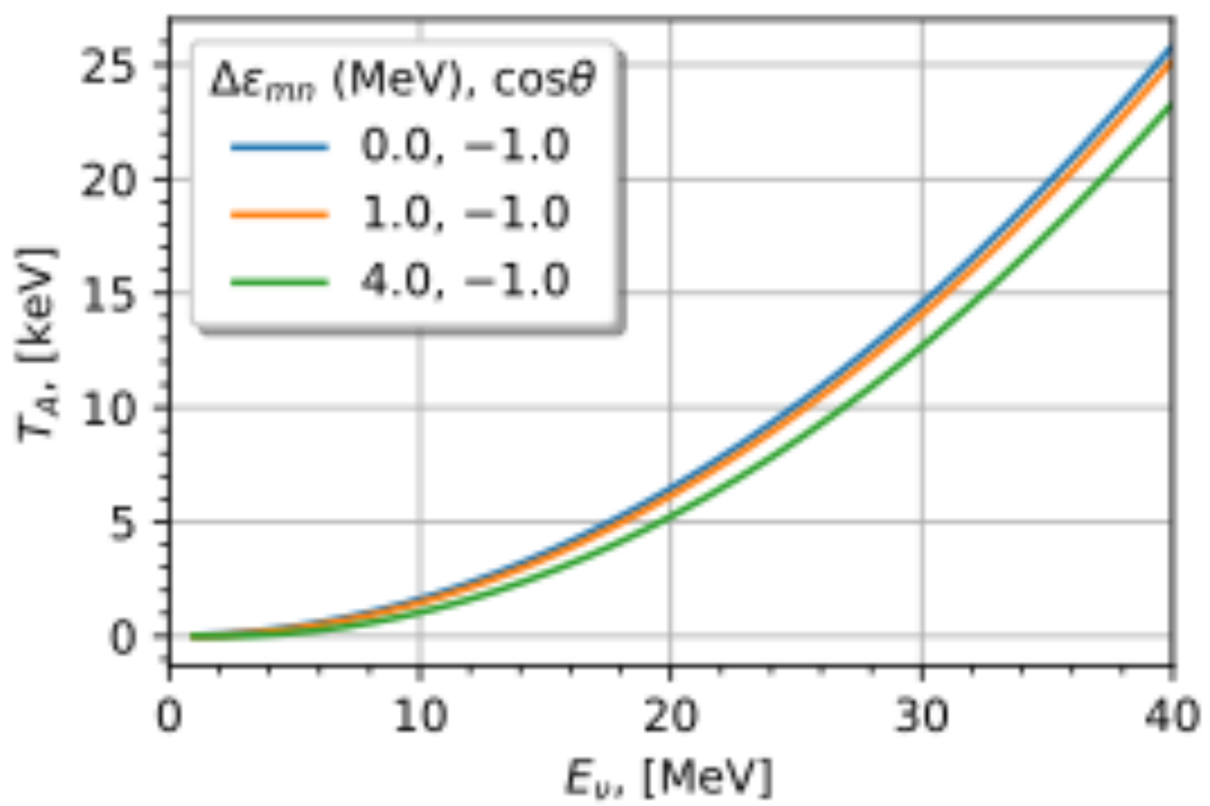
These are features opposite to 'coherent' scattering
Give rise to 'incoherent' scattering

Coherence and incoherence in elastic and inelastic neutrino-nucleus scattering

Derivation simplified for this lecture

If experiment is sensitive to differentiate an excited nucleus by its kinetic energy?





- No. The kinetic energy of a scattered nucleus is essentially the same
- The observable is a sum over all final states

The observable cross-section

$$\frac{d\sigma}{d\Omega} \propto \sum_m |\mathscr{A}_{mn}|^2 = |\mathscr{A}_0|^2 \sum_{k,j} \sum_m \left\langle n | e^{-i\mathbf{q}\mathbf{X}_\mathbf{j}} | m \right\rangle \left\langle m | | e^{+i\mathbf{q}\mathbf{X}_\mathbf{k}} | \right.$$

- Using

$$\sum_m |m\rangle\langle m| = \hat{I}$$

$$|\mathscr{A}|^2 = |\mathscr{A}_0|^2 \sum_{kj} \left\langle n | e^{-i\mathbf{q}\hat{\mathbf{X}}_\mathbf{j}} e^{i\mathbf{q}\hat{\mathbf{X}}_\mathbf{k}} | n \right\rangle .$$

- Considering terms with k=j and the rest terms

$$|\mathscr{A}|^2 = |\mathscr{A}_0|^2 \big(A + A(A-1)G(\mathbf{q})\big),$$

- where

$$G(\mathbf{q}) = \mathbf{A}^{-1}(\mathbf{A}-1)^{-1} \sum_{\mathbf{k} \neq \mathbf{j}} \left\langle \mathbf{n} | \mathbf{e}^{-\mathbf{iq}\hat{\mathbf{X}}_\mathbf{j}} \mathbf{e}^{\mathbf{iq}\hat{\mathbf{X}}_\mathbf{k}} | \mathbf{n} \right\rangle$$

Too many equations?
Are you tired?

You do not know what means to be tired

This is Kirill.
Kirill did not reach ...
He is tired a bit...

$$|\mathscr{A}|^2 = |\mathscr{A}_0|^2 \left(A + A(A-1) \underbrace{G(\mathbf{q})}_{elastic+inelastic} \right).$$

- $G(\mathbf{q})$ describes pair correlation. Elastic and inelastic contribute
- $\lim_{q\to\infty} G(\mathbf{q}) = 0$ and $|\mathscr{A}|^2 \to A|\mathscr{A}_0|^2$ (incoherent)
- In elastic only process $|\mathscr{A}|^2 = A^2|\mathscr{A}_0|^2|F(\mathbf{q})|^2$
- Thus, it is convenient

$$|\mathscr{A}|^2 = |\mathscr{A}_0|^2 \left(\underbrace{A^2|F(\mathbf{q})|^2}_{elastic} + \underbrace{A^2(G(\mathbf{q}) - |\mathbf{F}(\mathbf{q})|^2) + \mathbf{A}(1 - \mathbf{G}(\mathbf{q}))}_{inelastic} \right).$$

$$|\mathscr{A}|^2 = |\mathscr{A}_0|^2 \left(\underbrace{A^2 |F(\mathbf{q})|^2}_{elastic} + \underbrace{A^2(G(\mathbf{q}) - |\mathbf{F}(\mathbf{q})|^2) + \mathbf{A}(1 - \mathbf{G}(\mathbf{q}))}_{inelastic} \right).$$

- If pair correlations are ignored $G(\mathbf{q}) = |\mathbf{F}(\mathbf{q})|^2$

$$|\mathscr{A}|^2 = |\mathscr{A}_0|^2 \left(\underbrace{A^2 |F(\mathbf{q})|^2}_{elastic} + \underbrace{A(1 - |F(\mathbf{q})|^2)}_{inelastic} \right).$$

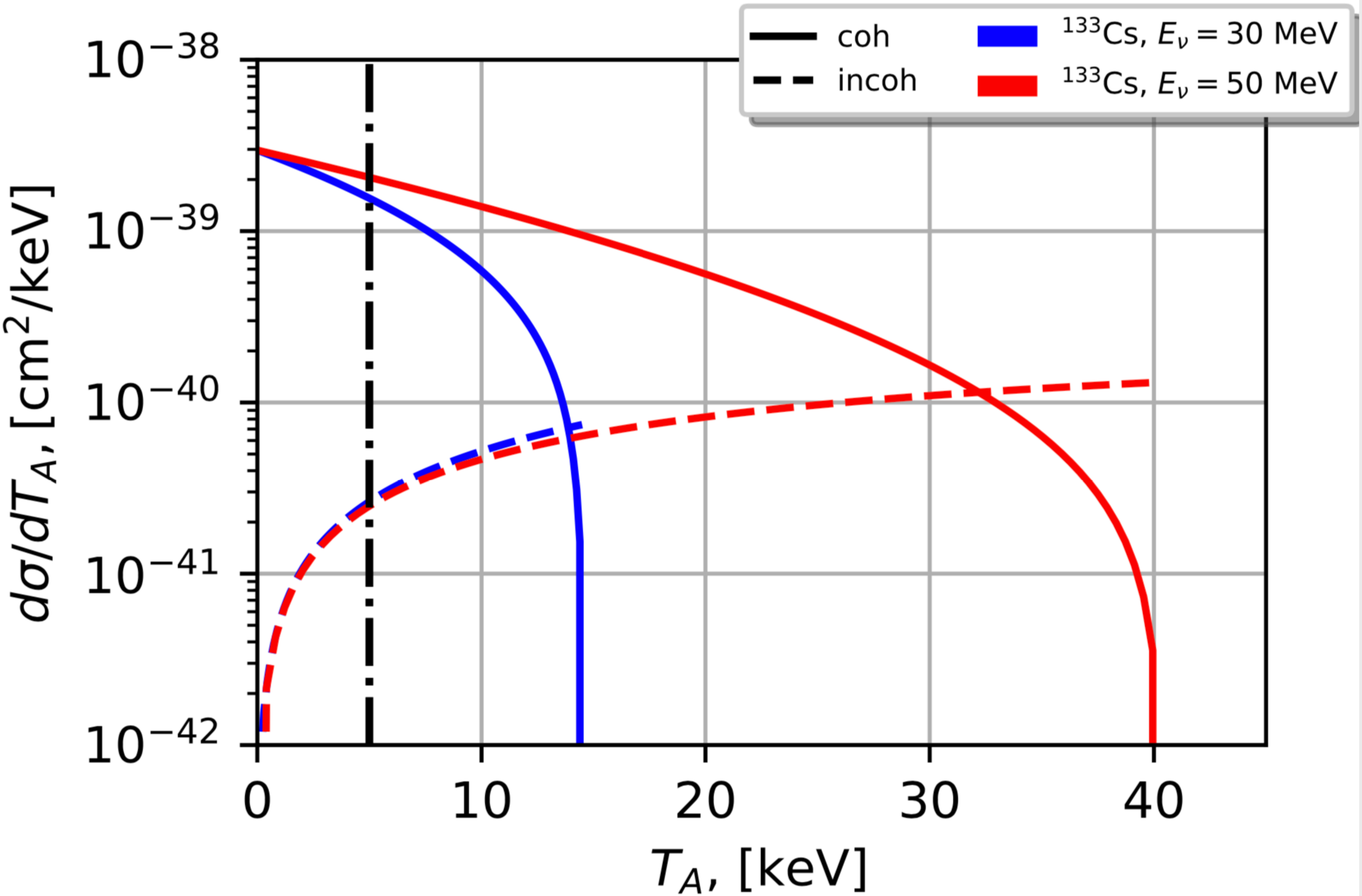
- This is quite general result illustrated in a simplified way
- The actual calculation
  - Within QFT framework of SM
  - Accounting for wave-functions of nucleons

Compare theory to experiment

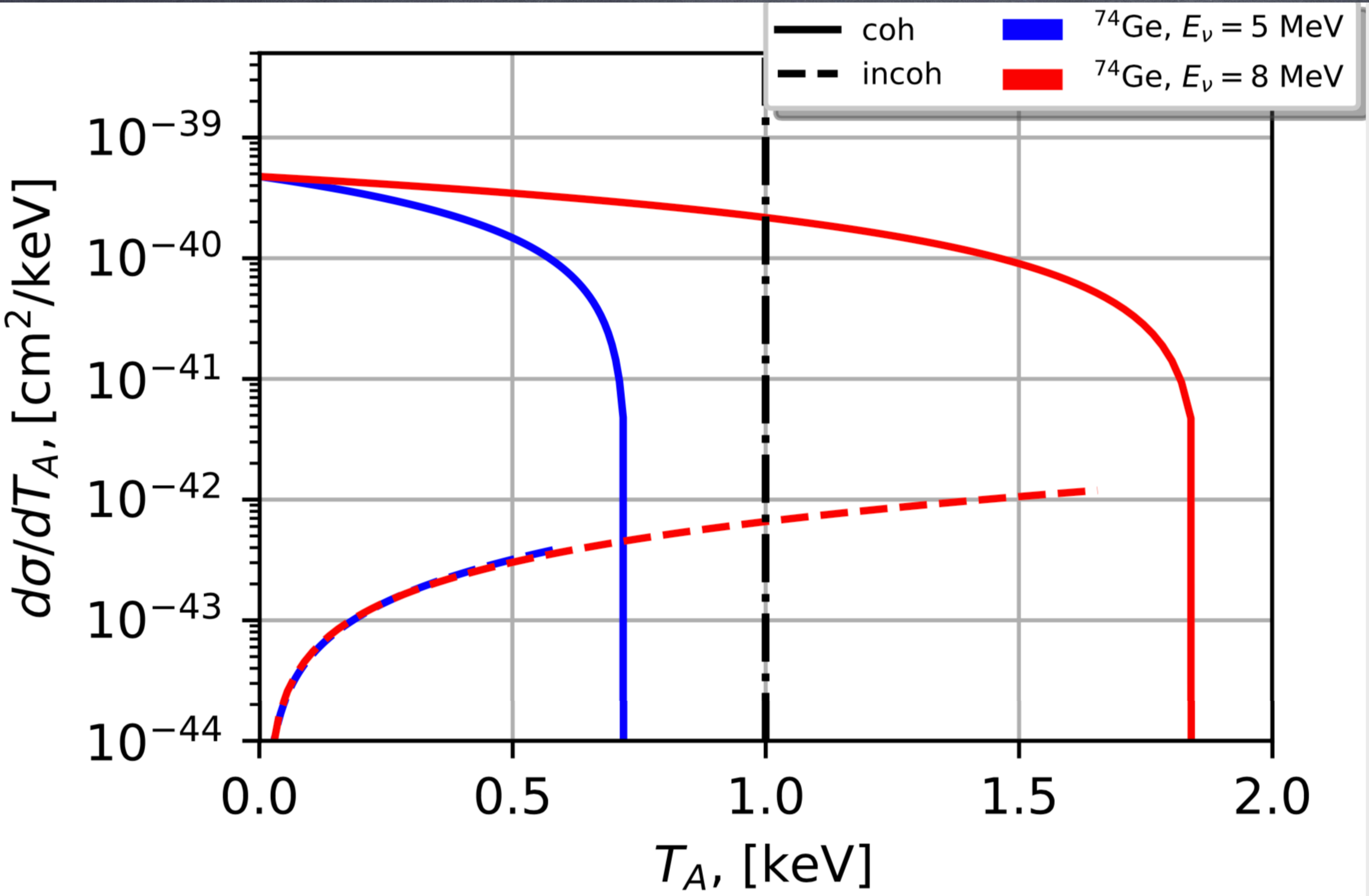
Two experimental setups

- COHERENT
  - E_ν=30-50 MeV
- nuGEN
  - Reactor anti-neutrino

Differential cross-section for COHERENT

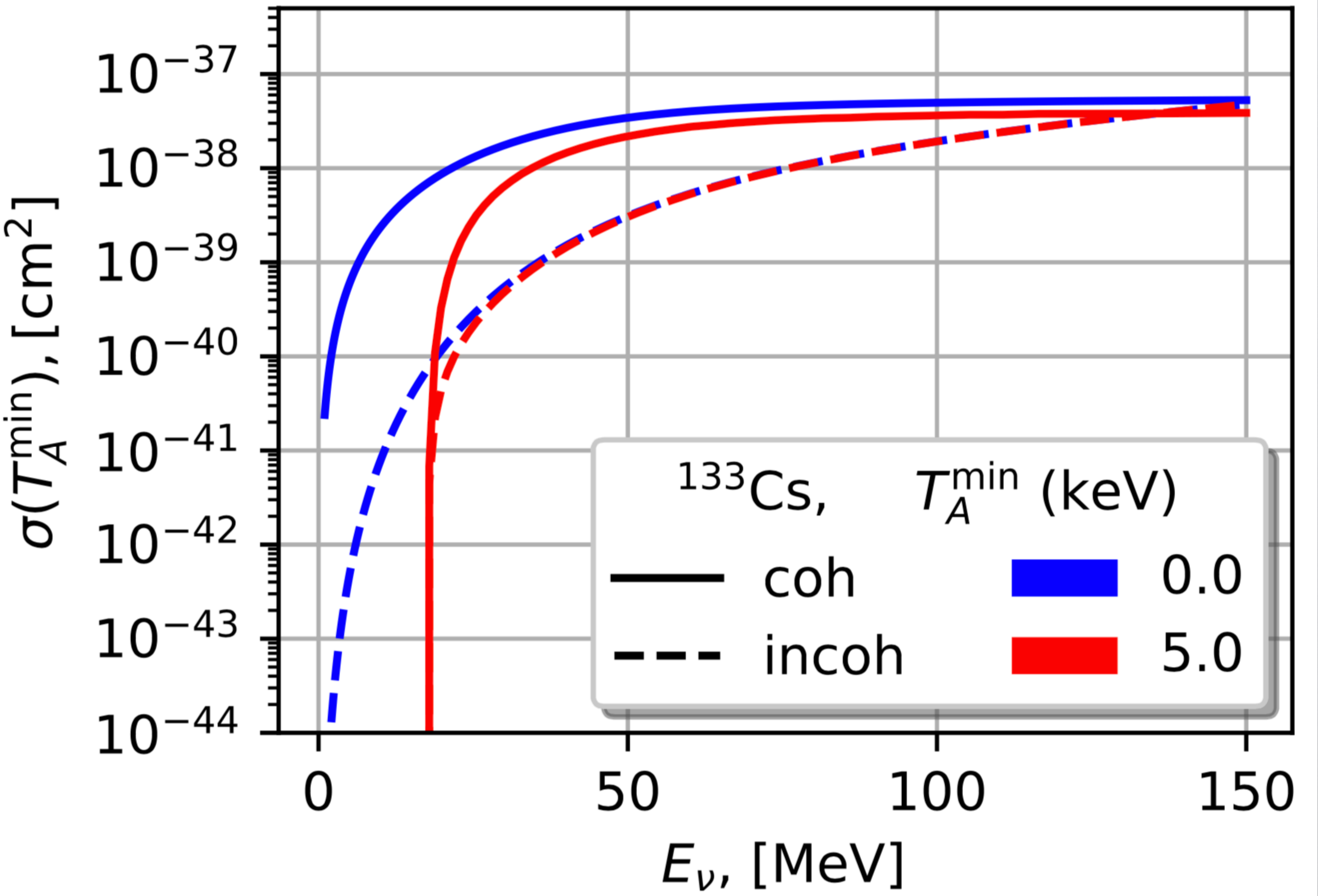


Differential cross-section for nuGEN

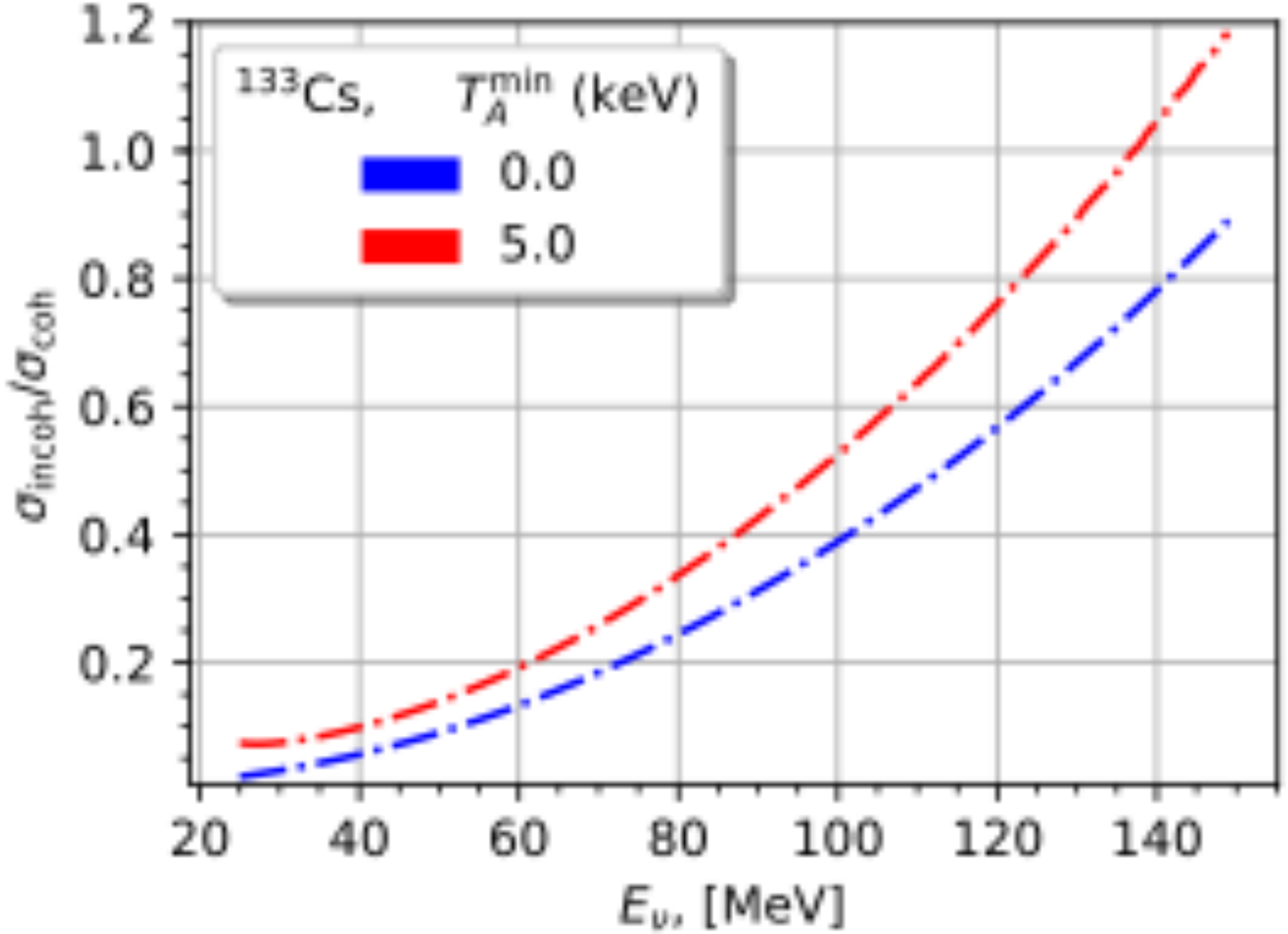


Integral Coherent and Incoherent Cross-sections

Integral cross-sections for COHERENT



Incoherent vs Coherent Cross-sections



^{133}Cs, T_A^{min} (keV)
0.0
5.0
1.2
1.0
0.8
0.6
0.4
0.2
$\sigma_{incoh}/\sigma_{coh}$
20
40
60
80
100
120
140
E_ν, [MeV]

Short summary

- Now we do have a theory of neutrino-nucleus scattering with appropriate coherent and incoheren regimes
- Incoherent scattering is of importance for

$$E_\nu \geq 30\text{MeV}$$

- Coherent and incoherent is not very accurate terminology
- Better to talk about elastic and inelastic, quadratic and linear as the number of nucleons

QFT derivation

In all details

observable differential cross-section defined in Eq. (42)

$$\frac{d\sigma}{dT_A} = \frac{G_F^2 m_A}{2^6\pi m_N^2 E_\nu^2}$$
$$\times \sum_{k,j=1}^{A}\sum_n \omega_n C_{1,mn}C_{2,mn}\Bigg(f^k_{nn} f^{j*}_{nn} \sum_r (l, h^k_{rr}) \sum_s (l, h^j_{ss})^\dagger$$
$$+ \sum_{m\neq n} f^k_{mn} f^{j*}_{mn} \sum_{sr}\lambda^{mn}_{sr}(l, h^k_{sr}) \Big(\sum_{s'r'}\lambda^{mn}_{s'r'}(l, h^j_{s'r'})\Big)^\dagger\Bigg) \quad \text{(B24)}$$

expressed through the scalar products $(l, h^{p/n}_{sr})$ of 4-vectors with components $l^\mu(k,k')$ given by Eq. (B4) and

$$(h^{p/n}_{sr})_\mu = \overline{u}(\boldsymbol{p}+\boldsymbol{q}, s) O^{p/n}_\mu u(\boldsymbol{p}, r) \quad \text{(B25)}$$

where $\boldsymbol{p}$ is a solution of Eq. (32). In Eq. (B25) a superscript p or n appears when the index k in h^k_{sr} from Eq. (B24) points to a proton or to a neutron, respectively.

When an index k or j in Eq. (B24) points to a proton/neutron, the form-factors f^k_{mn} should be read as $f^{p/n}_{mn}$, correspondingly.

Each of the $|(l, h^{p/n})|^2$ terms given by Eqs. (C12) and (C33) yields the common factor $64(s-m_N^2)^2$, where $s = (p+k)^2$ is the total energy squared in the neutrino-nucleon center-of-mass frame, and m_N is the mass of the nucleon. In the leading non-relativistic approximation this factor can be approximated as $2^8 m_N^2 E_\nu^2$. We denote a correction to this formula by a factor $C_{3,mn}$, accounting for the fact that the nucleon in the initial state has a non-zero three-momentum

$$(s-m_N^2)^2 = 4m_N^2 E_\nu^2 C_{3,mn}. \quad \text{(B26)}$$

In what follows we denote by g^{mn} the product of correction factors

$$g^{mn} = C_{1,mn}C_{2,mn}C_{3,mn} \quad \text{(B27)}$$

which is of the order of unity.

Following our discussion of Eq. (37) we identify the second and third lines of Eq. (B24) as contributing to the coherent and incoherent cross-sections. The factor g^{mn} is, in general, different for coherent and incoherent terms. We take out these factors from the double summation at their effective

and

Let us work out the incoherent scattering encoded in the third line of Eq. (B24). A summation over m, n cannot be done without a model for λ^{mn}_{sr}. If λ^{mn}_{sr} would not depend on m, n the corresponding summation could be performed as follows.

Consider the case when k and j point to the same type of the nucleon, for example, to a proton.

If $k = j$, then

$$\sum_n \omega_n \sum_{m\neq n} f^k_{mn} f^{k*}_{mn} = \sum_n \omega_n \Big[\sum_m f^k_{mn} f^{k*}_{mn} - f^k_{nn} f^{k*}_{nn}\Big]$$
$$= \sum_n \omega_n \Big[\langle n|e^{-i\boldsymbol{q}\boldsymbol{X}_k}\sum_m |m\rangle\langle m|e^{i\boldsymbol{q}\boldsymbol{X}_k}|n\rangle\Big] - |F_p(\boldsymbol{q})|^2$$
$$= 1 - |F_p(\boldsymbol{q})|^2, \quad \text{(B30)}$$

accounting for the equality $\sum_m |m\rangle\langle m| = \hat{I}$, using Eq. (B28) and normalizations in Eq. (A23) and $\sum_n \omega_n = 1$.

If $k \neq j$ then following a consideration similar to Eq. (B30) one may find that

$$\sum_n \omega_n \sum_{m\neq n} f^k_{mn} f^{j*}_{mn} = \langle \mathrm{cov}(e^{-i\boldsymbol{q}\hat{\boldsymbol{X}}_j}, e^{i\boldsymbol{q}\hat{\boldsymbol{X}}_k})\rangle_p \quad \text{(B31)}$$

where the right-hand-side of Eq. (B31) is a covariance of quantum operators $e^{-i\boldsymbol{q}\hat{\boldsymbol{X}}_j}$ and $e^{i\boldsymbol{q}\hat{\boldsymbol{X}}_k}$ on $|n\rangle$, whose state reads

$$\mathrm{cov}_{nn}(e^{-i\boldsymbol{q}\hat{\boldsymbol{X}}_j}, e^{i\boldsymbol{q}\hat{\boldsymbol{X}}_k})$$
$$= \langle n|e^{-i\boldsymbol{q}\hat{\boldsymbol{X}}_j} e^{i\boldsymbol{q}\hat{\boldsymbol{X}}_k}|n\rangle - \langle n|e^{i\boldsymbol{q}\hat{\boldsymbol{X}}_k}|n\rangle\langle n|e^{-i\boldsymbol{q}\hat{\boldsymbol{X}}_j}|n\rangle. \quad \text{(B32)}$$

The subscript p in Eq. (B31) refers to a proton.

The averaging $\langle\ldots\rangle$ in Eq. (B31) is given by

$$\langle \mathrm{cov}(e^{-i\boldsymbol{q}\hat{\boldsymbol{X}}_j}, e^{i\boldsymbol{q}\hat{\boldsymbol{X}}_k})\rangle_p = \sum_n \omega_n \mathrm{cov}_{nn}(e^{-i\boldsymbol{q}\hat{\boldsymbol{X}}_j}, e^{i\boldsymbol{q}\hat{\boldsymbol{X}}_k}). \quad \text{(B33)}$$

At both, $\boldsymbol{q} \to 0$ and $\boldsymbol{q} \to \infty$

$$\lim_{\boldsymbol{q}\to 0}\langle \mathrm{cov}(e^{-i\boldsymbol{q}\hat{\boldsymbol{X}}_j}, e^{i\boldsymbol{q}\hat{\boldsymbol{X}}_k})\rangle_p = 0,$$
$$\lim_{\boldsymbol{q}\to \infty}\langle \mathrm{cov}(e^{-i\boldsymbol{q}\hat{\boldsymbol{X}}_j}, e^{i\boldsymbol{q}\hat{\boldsymbol{X}}_k})\rangle_p = 0. \quad \text{(B34)}$$

) can be rewritten, factoring out the

ν and the factor $C_{mn,1}$ of the order

$$\frac{P^0_n}{E_p}\frac{P'^0_m}{E_{p+q}}\Big)^{1/2}\frac{m_N}{m_A}. \quad \text{(B19)}$$

7) and (B19), one can repre-

7).

ross-sections

esponding to the matrix element

$$\frac{E_\nu - E'_\nu - T_A - \Delta\varepsilon_{mn})}{m_A + T_A + \varepsilon_m}, \quad \text{(B20)}$$

es are given in the laboratory frame

ıs is assumed to be at rest, E'_ν is

$_{mn} = \varepsilon_m - \varepsilon_n$. The kinetic energy

ıs is given by Eqs. (26) and (27).

be done with help of a Dirac δ-

conservation, thus yielding

$$\frac{E'_\nu(m_A+T_A)}{\varepsilon_n)\,E_\nu(m_A+T_A+\varepsilon_m)} \quad \frac{1}{-\cos\theta) - \Delta\varepsilon_{mn}}, \quad \text{(B21)}$$

using a very accurate approxima-

Kinematic Paradox

- Coherent scattering is essentially an elastic process
- The nucleus remains in the same state
- Neutrino transfers 3-momentum q to the nucleus. What is kinetic energy of the nucleus?

$$T_A = \frac{\mathbf{q}^2}{2M_A}$$

- But first neutrino transfers 3-momentum q to a nucleon assumed to be at rest. What is kinetic energy of the nucleon?

$$T_N = \frac{\mathbf{q}^2}{2M_N} \text{factor} \frac{M_N}{M_A} \text{larger}$$

- The nucleon can not change its potential energy because the entire nucleus remains in the same quantum state.
- So, we have a violation of energy conservation

$$\text{potential energy} + \frac{\mathbf{q}^2}{2M_N} \neq \text{potential energy} + \frac{\mathbf{q}^2}{2M_A}$$

- What is wrong?

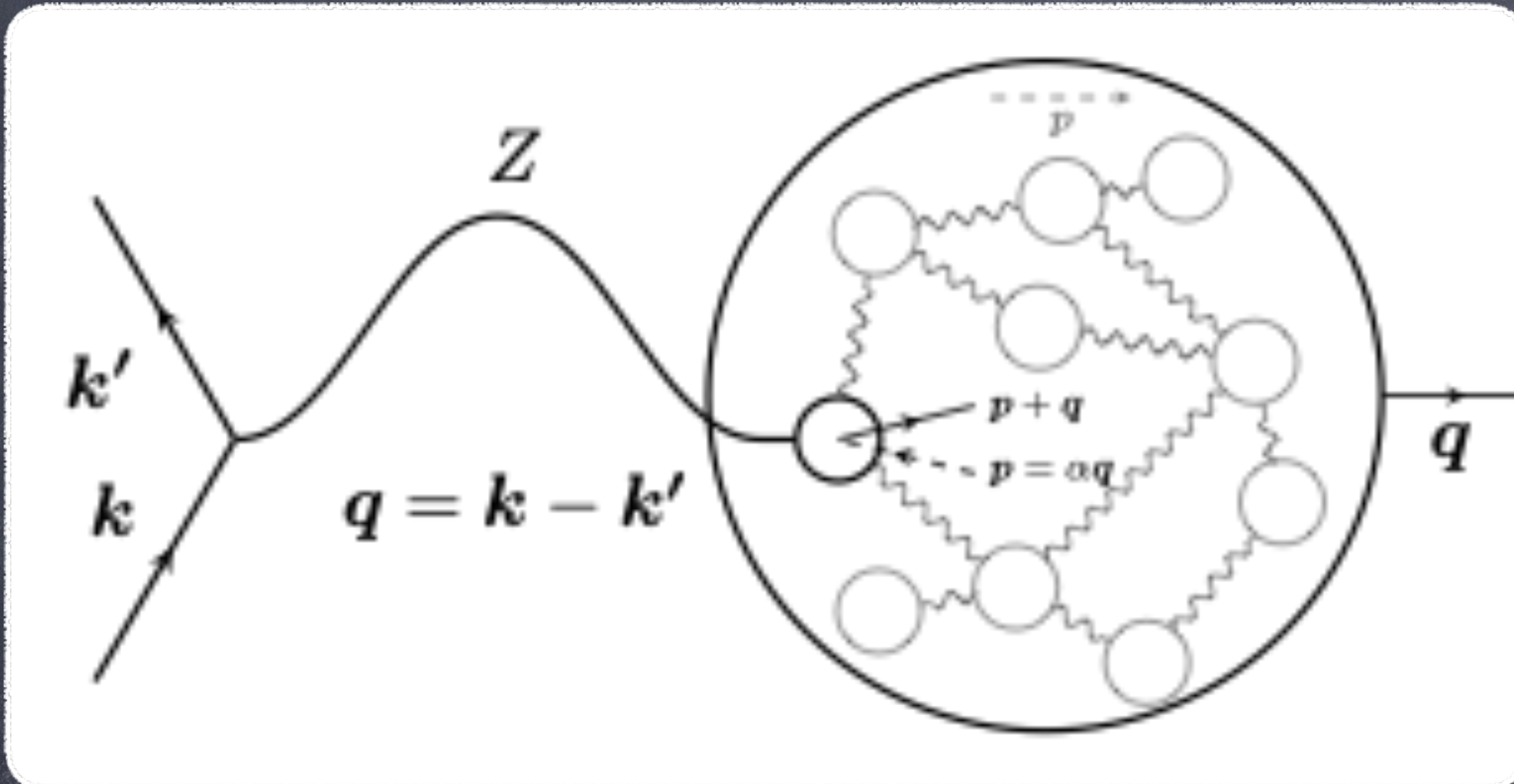
What is wrong?

- One of the assumptions must be wrong.
- We assumed a target nucleon to be at rest. It seems reasonable but this leads to the paradox.
- Which 3-momentum of target nucleon is appropriate to
  - Conserve energy-momentum
  - Keep nucleus in the same quantum state

$$\frac{(\mathbf{p}+\mathbf{q})^2}{2M_N} - \frac{\mathbf{p}^2}{2M_N} = \frac{\mathbf{q}^2}{2M_A}$$

- The target nucleon momentum

$$\mathbf{p} = -\frac{\mathbf{q}}{2}\left(1 - \frac{\mathbf{M_N}}{\mathbf{M_A}}\right)$$



- Not any nucleon can interact with neutrino to keep the nucleus in the same state
- Find a larger momentum in nucleus is less probable. Mathematically this leads to the form-factor $|F(\mathbf{q})|^2$

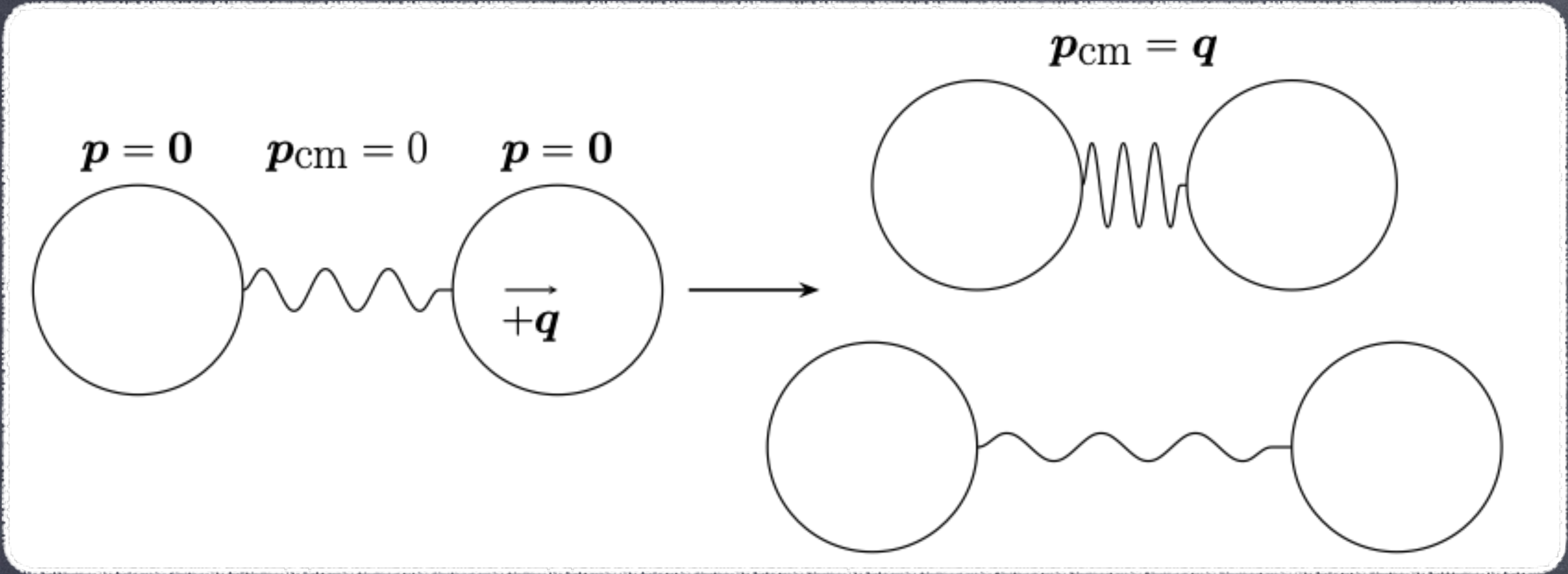
Additional matter

De-excitation gammas

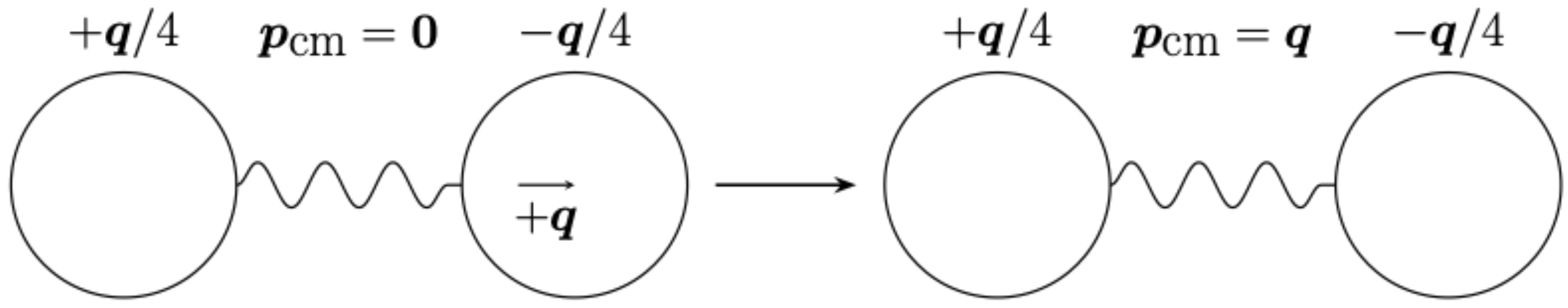
Inelastic interactions

- Produce nucleus in an excited state
- De-excitation of a nucleus often releases gammas which could be detected
- Detection of de-excitation gammas could help to constrain the nucleus form-factor and more accurately measure the elastic part.

Example with two balls connected with a spring



Two balls get excited after an interaction.
Inelastic scattering



elastic scattering

Before:

$$\left(\frac{q}{4}\right)^2 \frac{1}{2m} + \left(\frac{q}{4}\right)^2 \frac{1}{2m} = \frac{q^2}{16m}$$

After:

Total:

$$\left(\frac{q}{4}\right)^2 \frac{1}{2m} + \left(\frac{3q}{4}\right)^2 \frac{1}{2m} = \frac{5q^2}{16m}$$

Center-of-mass energy:

$$\frac{q^2}{4m}$$

Potential:

$$\frac{5q^2}{16m} - \frac{q^2}{4m} = \frac{q^2}{16m}$$

$$
\begin{aligned}
f^k_{mn}(\mathbf{q}) &= \left\langle \mathbf{m} \,|\, e^{i\mathbf{q}\widehat{\mathbf{X}}_{\mathbf{k}}} \,|\, \mathbf{n} \right\rangle \\
&= \int (\prod_{i=1}^{A} d\mathbf{x}_{\mathbf{i}}) \psi^*_{\mathbf{m}}(\mathbf{x}_1 \ldots \mathbf{x}_{\mathbf{A}}) \psi_{\mathbf{n}}(\mathbf{x}_1 \ldots \mathbf{x}_{\mathbf{A}}) e^{i\mathbf{q}\mathbf{x}_{\mathbf{k}}},
\end{aligned}
$$