LIGO

Detection techniques for gravitational waves

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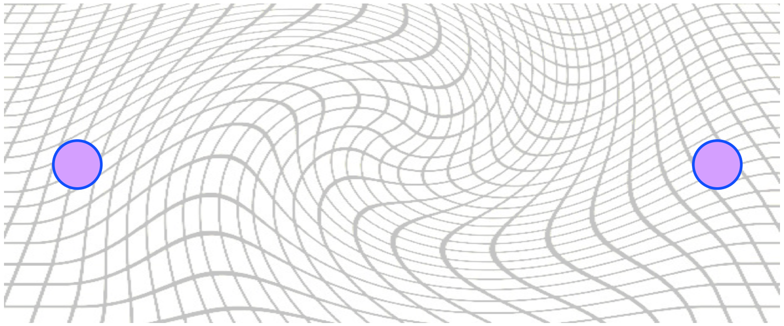
Overview

- Detection of gravitational waves
- Optical interferometers
- Noise sources in the LIGO detectors
- Future prospects

LIGO

Free masses



The distance between the two points is determined by the metric

$$ds^2 = \sum_{uv} g_{uv}(x,t)dx^u dx^v$$

The metric is determined by the Einstein equations

$$R_{ik} = -\frac{8\pi G}{c^4}(T_{ik} - \frac{1}{2}g_{ik}T^m{}_m)$$

$R_{ik}(g_{ik})$ is the Ricci tensor

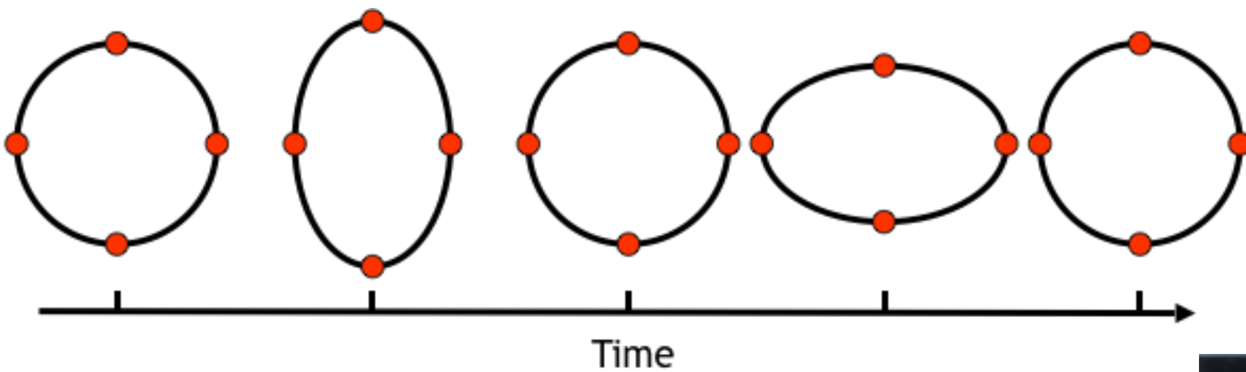
T_{ik} is the energy-momentum tensor

Free masses

In the weak field regime

$$g_{uv} = \mathrm{diag}(1, -1, -1, -1) + h_{uv}, \;\; h_{uv} \ll 1$$

And Einstein equations give a wave equation $\Box h_{uv} = 0$



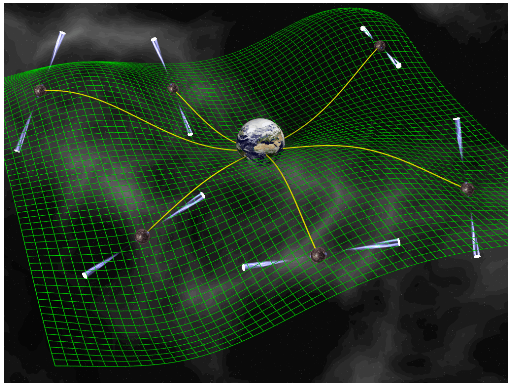
The strength of the signal is determined by the quadruple moment of the source I and distance from the source r

$$h_{uv}(t) = \frac{2G}{rc^4}\ddot{I}_{uv}(t - \frac{r}{c})$$

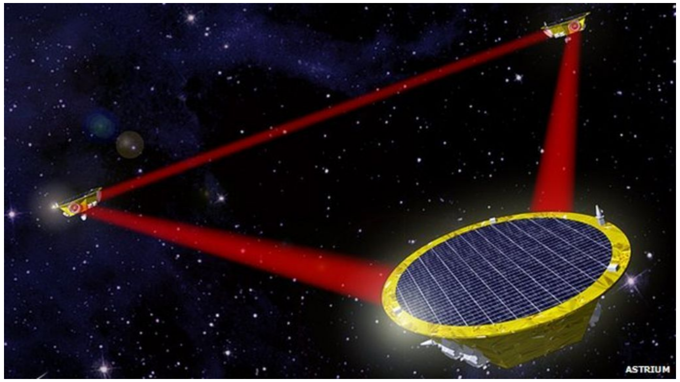
LIGO

Measurement of the fluctuations

- Pulsars and Earth (Nanograv)



- Free masses in space (LISA)



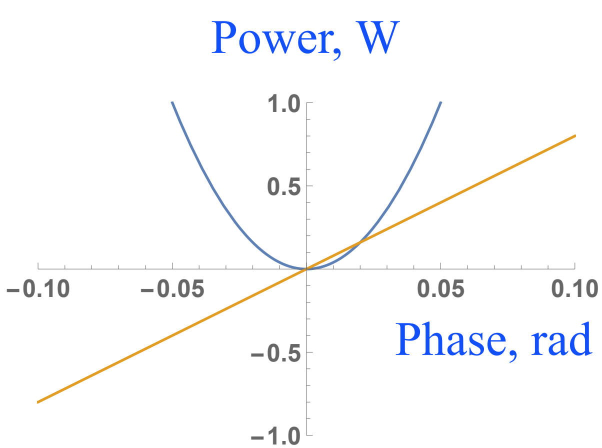
- Free masses on Earth (LIGO)

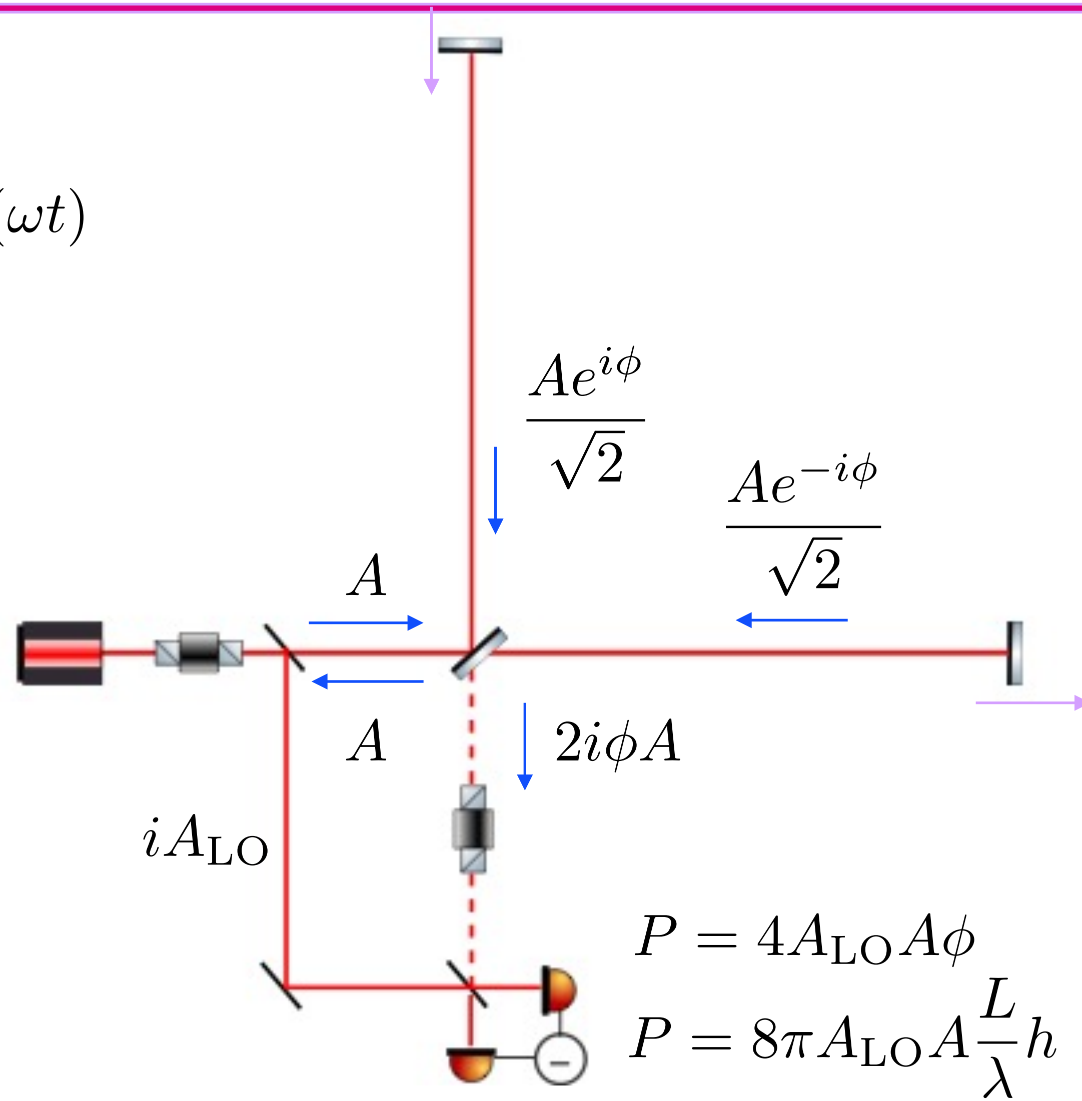
Michelson interferometer

Input laser field

$$E = E_1 \sin(\omega t) + E_2 \cos(\omega t)$$
$$= A e^{-i\omega t} + c.c.$$

For the best sensitivity:
Maximize L and A

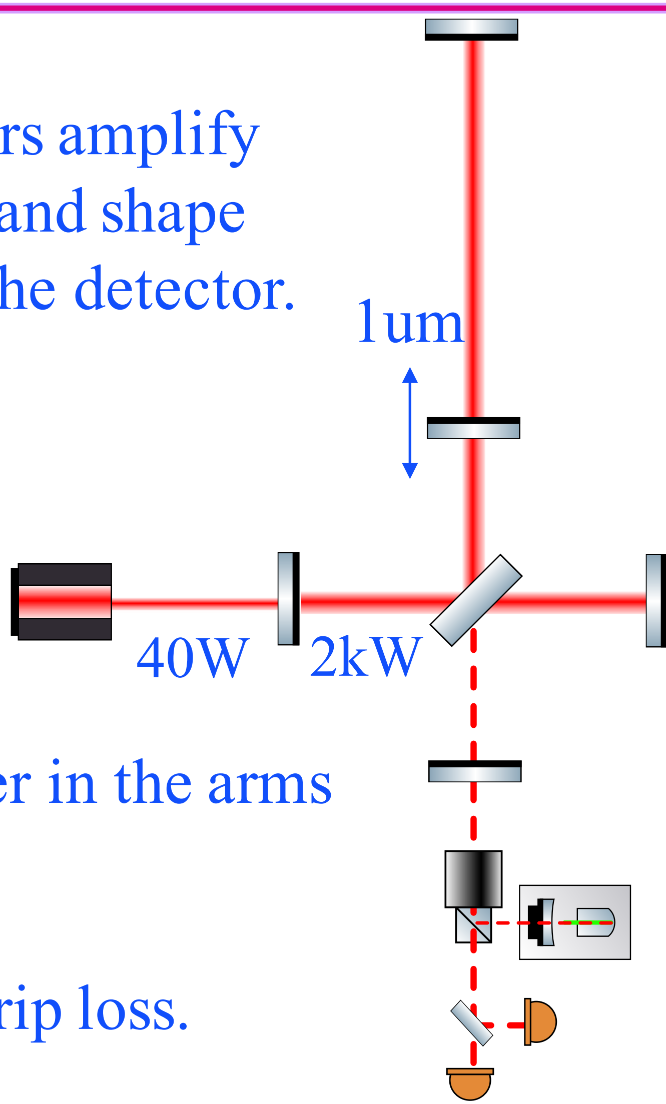


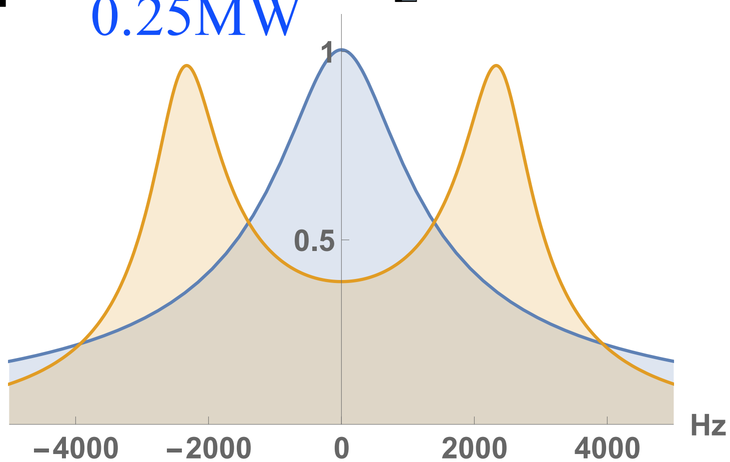


$$P = 4 A_{\mathrm{LO}} A \phi$$
$$P = 8\pi A_{\mathrm{LO}} A \frac{L}{\lambda} h$$

LIGO optical layout

Optical resonators amplify the input power and shape the response of the detector.



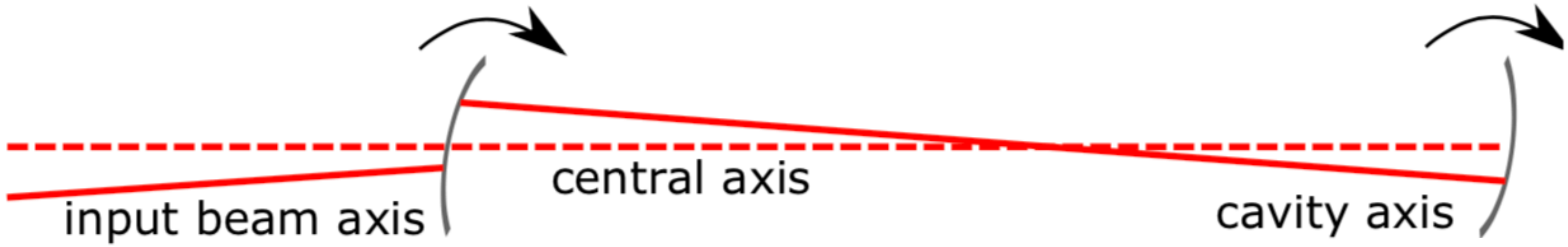


Maximum power in the arms

$$P_{\text{arm}} = \frac{P_{\text{laser}}}{2Y}$$

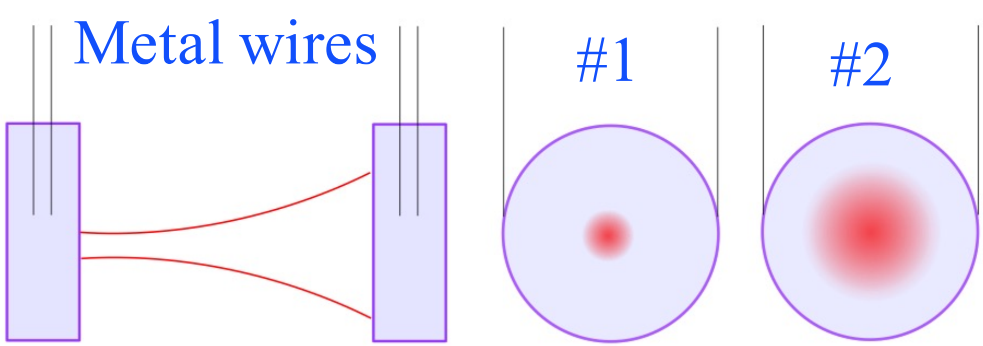
Y is the round trip loss.

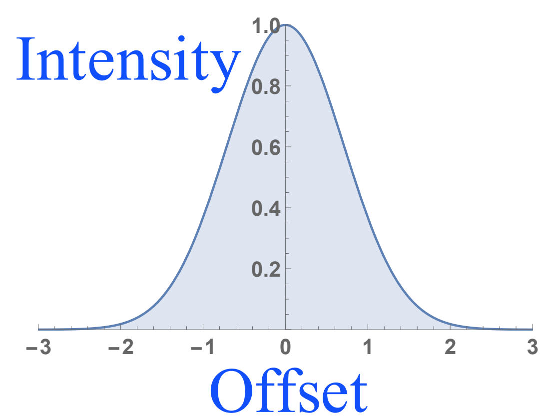
Angular motion



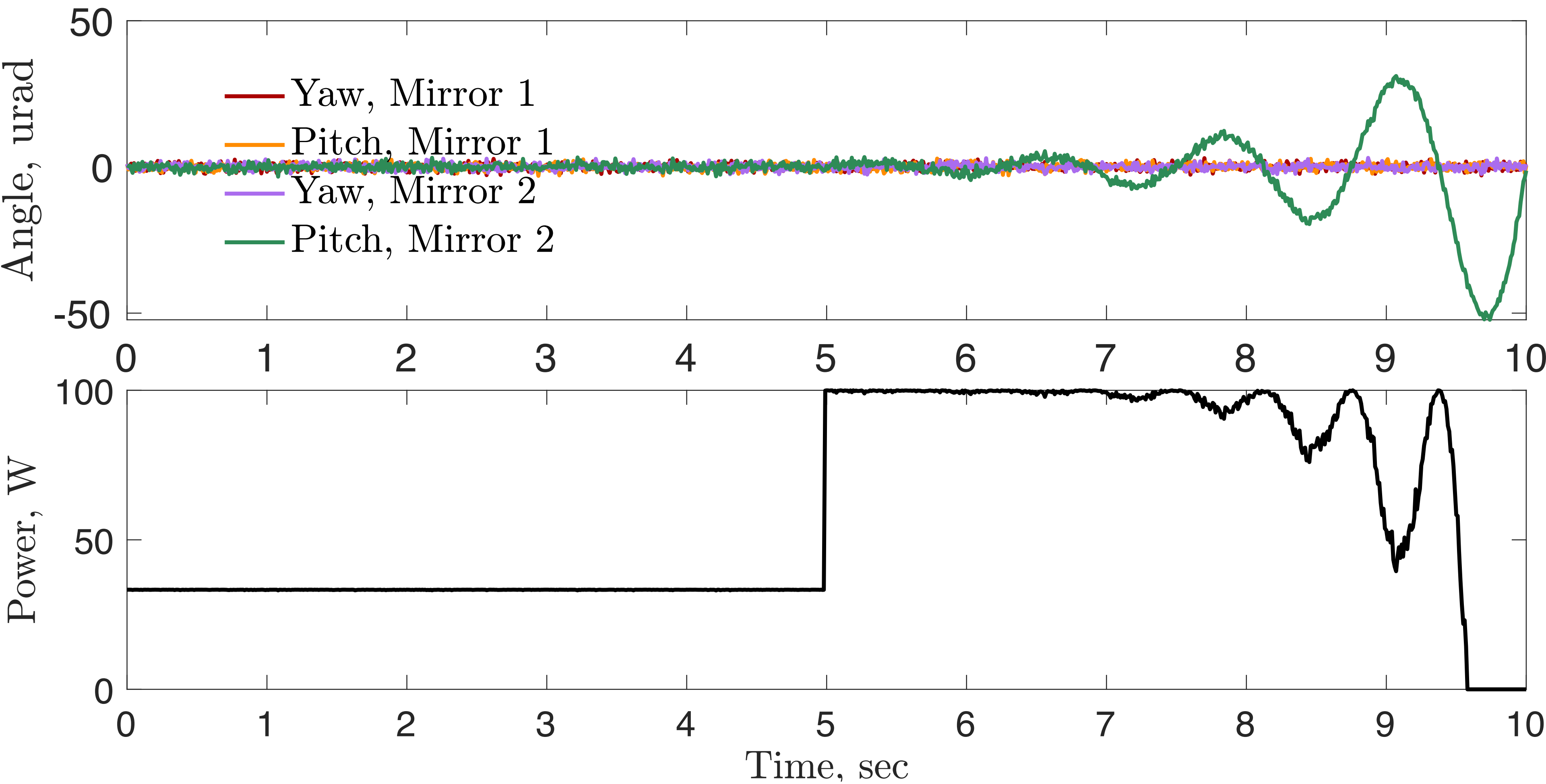
Resonating power

$$P \approx P_0 \left(1 - \left(\frac{\theta_{1,\mathrm{yaw}}}{\theta_1} \right)^2 - \left(\frac{\theta_{1,\mathrm{pitch}}}{\theta_1} \right)^2 - \left(\frac{\theta_{2,\mathrm{yaw}}}{\theta_2} \right)^2 - \left(\frac{\theta_{2,\mathrm{pitch}}}{\theta_2} \right)^2 \right)$$





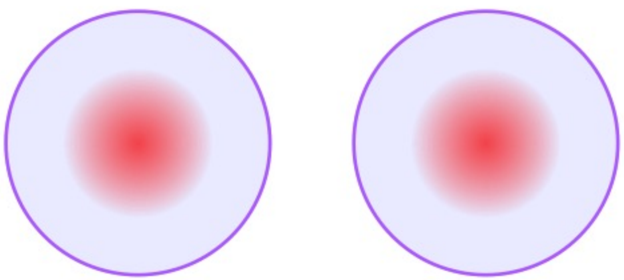
Instabilities



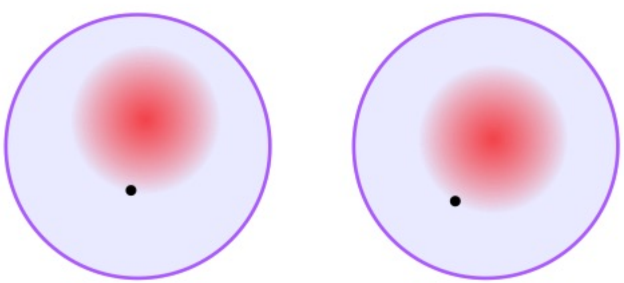
High power effects

- Beam intensity is 7 kW / cm^2 << 1 MW / cm^2
- The key problem comes from point absorbers on the mirrors

Input test masses



End test masses



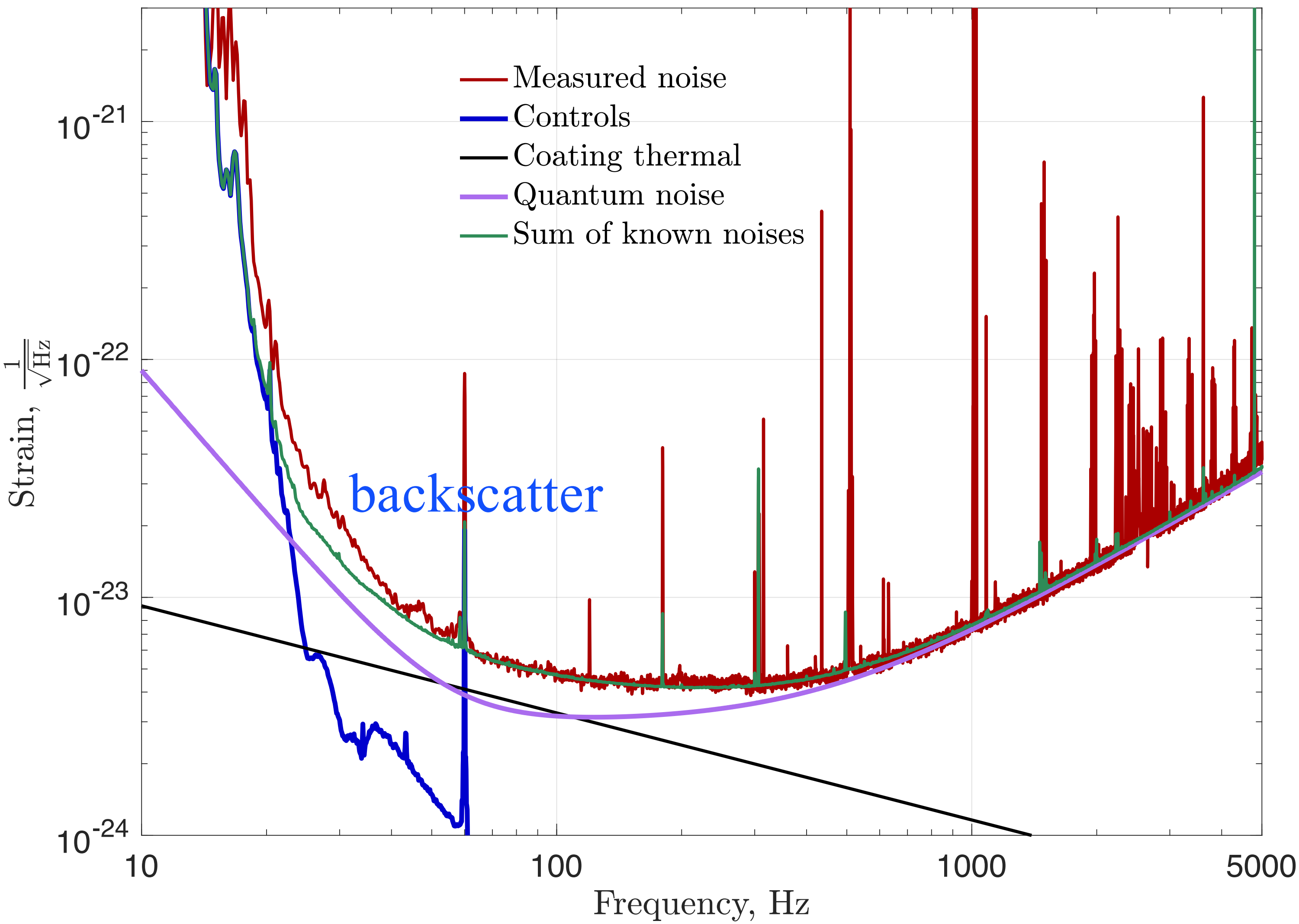
Maximum power in the arms

$$P_{\rm arm} = \frac{P_{\rm laser}}{2Y(P_{\rm arm})}$$

Temperature of the absorber

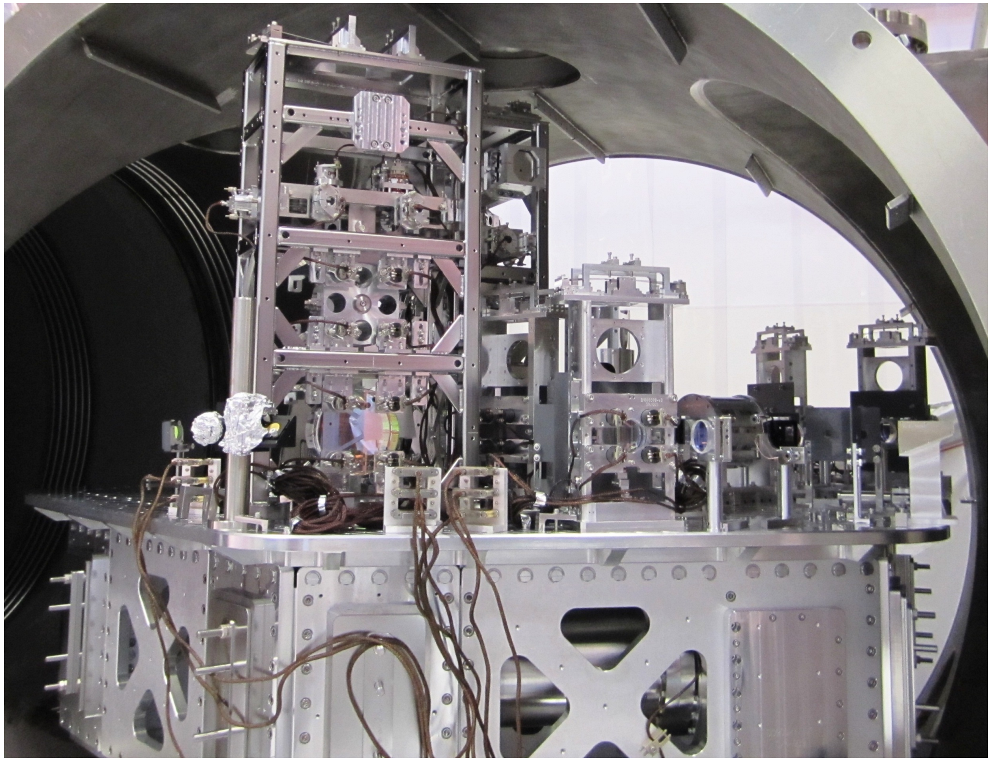
$$T = \frac{\beta I w}{2\kappa}$$

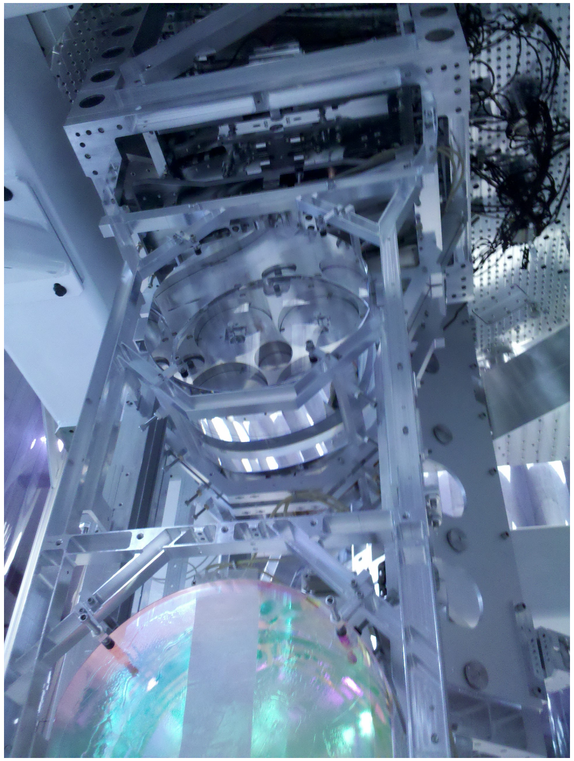
Current sensitivity



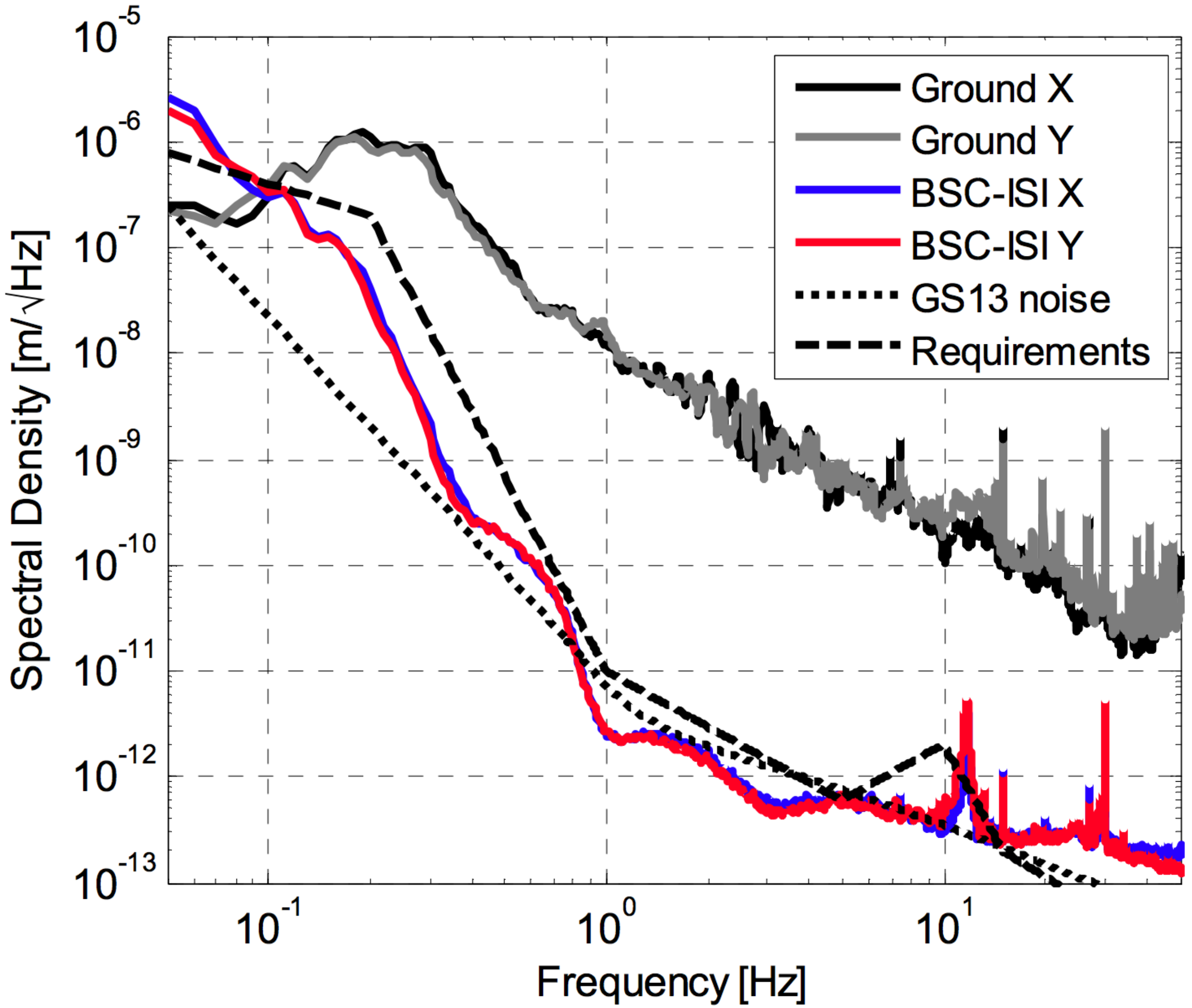
Ground vibrations

- Two types of isolation: active and passive

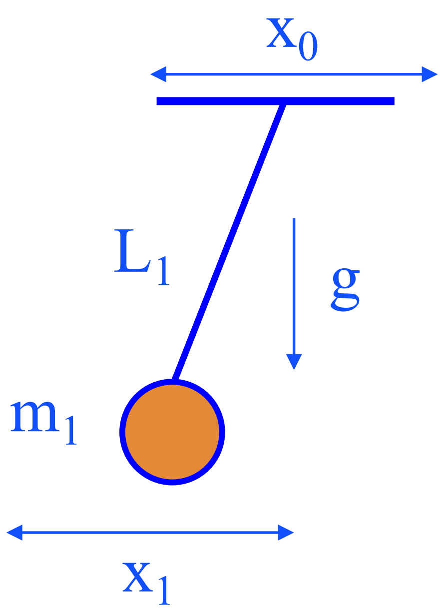




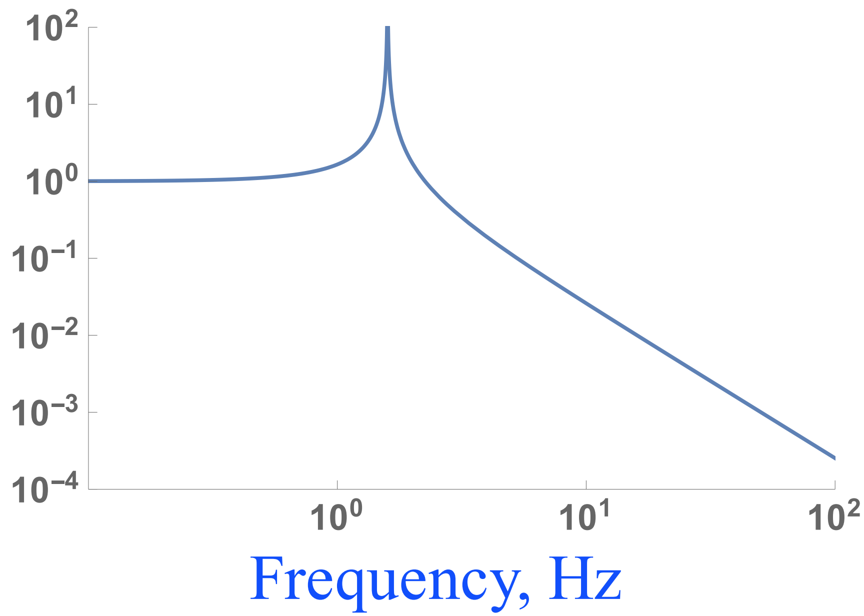
Active isolation



Passive isolation



Transfer function of vibrations to the test masses

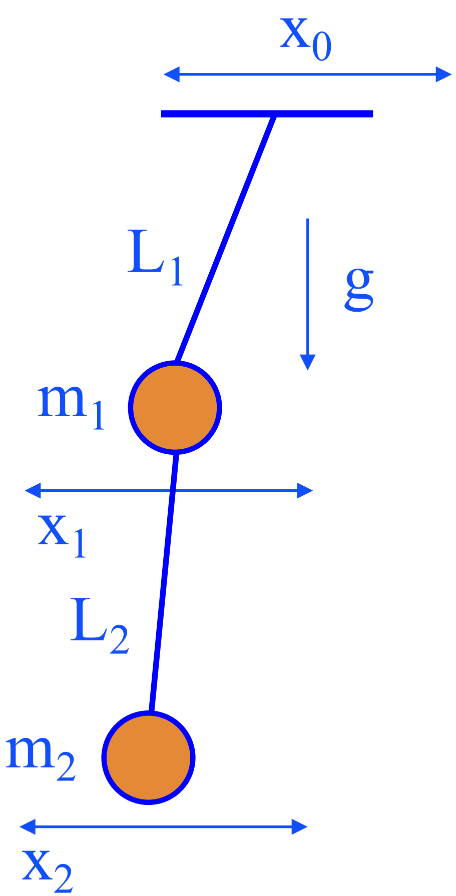


$$\mathcal{L} = \frac{m_1 \dot{x_1}^2}{2} - \frac{m_1 g L_1}{2} \left(\frac{x_1 - x_0}{L_1} \right)^2$$

$$\ddot{x} + \frac{g}{L_1} x = \frac{g}{L_1} x_0$$

$$x(\Omega) = \frac{\Omega_0^2}{-\Omega^2 + \Omega_0^2} x_0(\Omega) \underset{\Omega \gg \Omega_0}{\approx} -\frac{\Omega_0^2}{\Omega^2} x_0(\Omega)$$

Passive isolation



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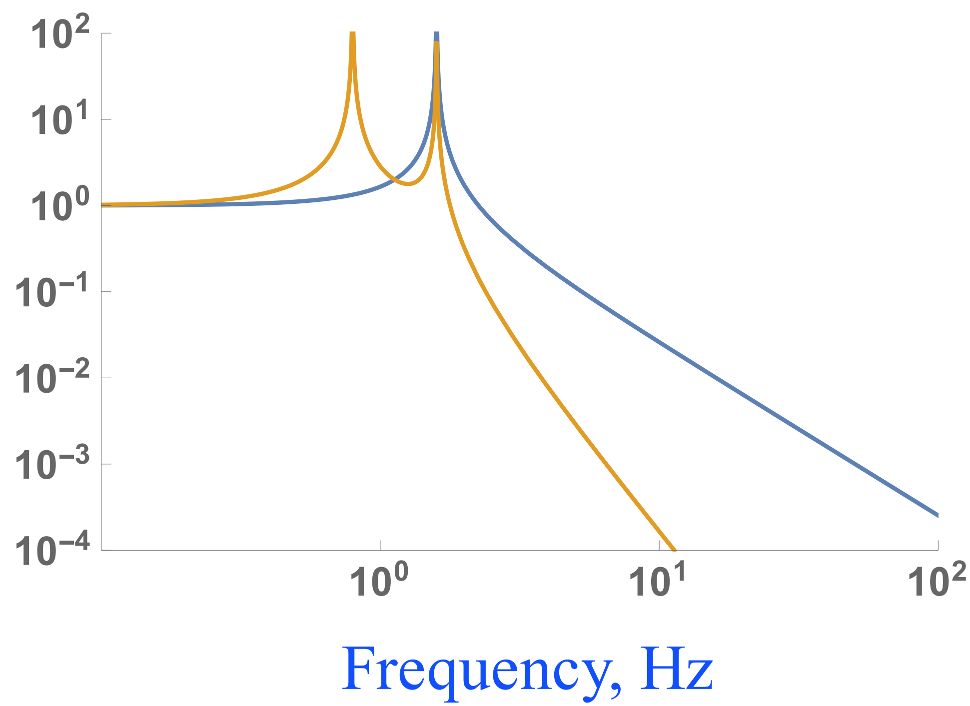
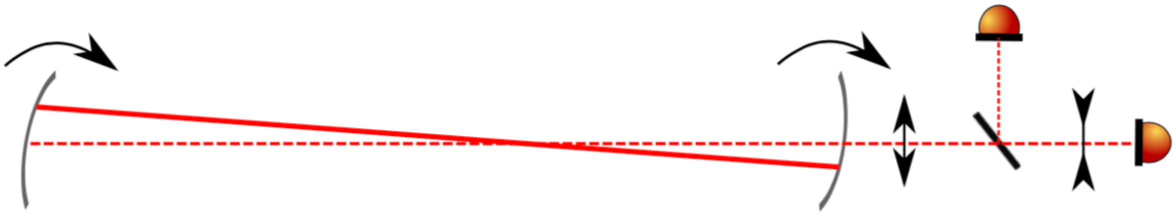
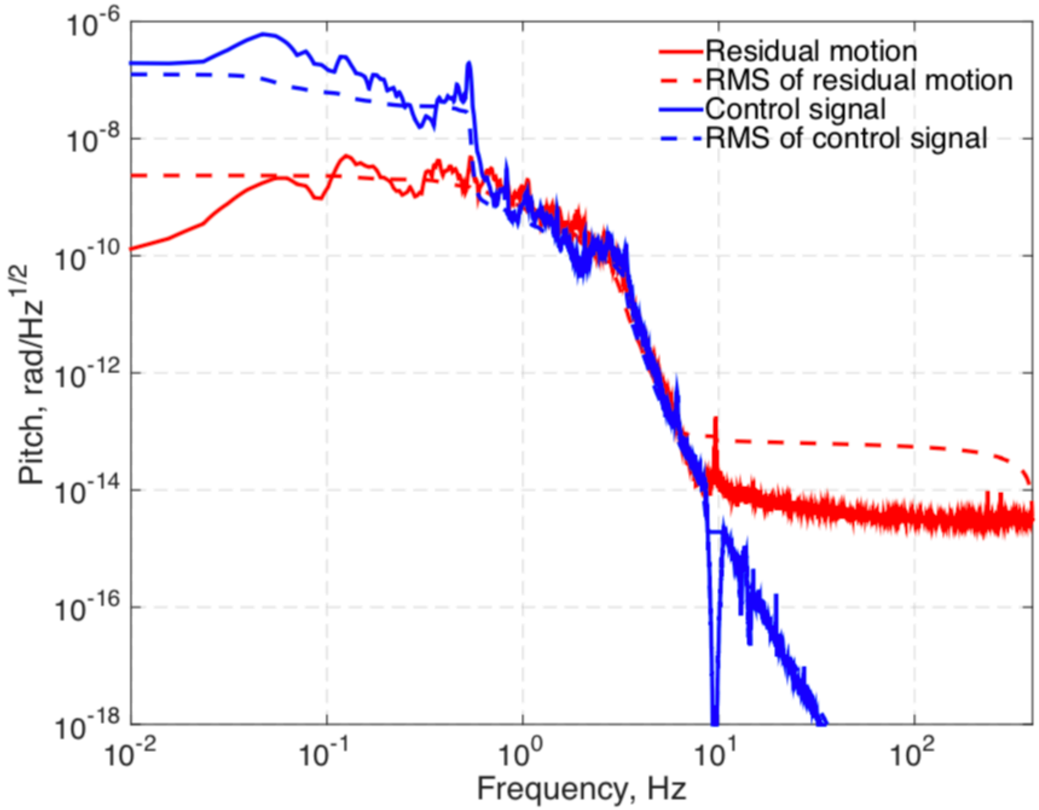


$$\mathcal{L} = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2^2}{2} - \frac{(m_1 + m_2) g L_1}{2} \left(\frac{x_1 - x_0}{L_1} \right)^2 - \frac{m_2 g L_2}{2} \left(\frac{x_2 - x_1}{L_2} \right)^2$$

Controls noise

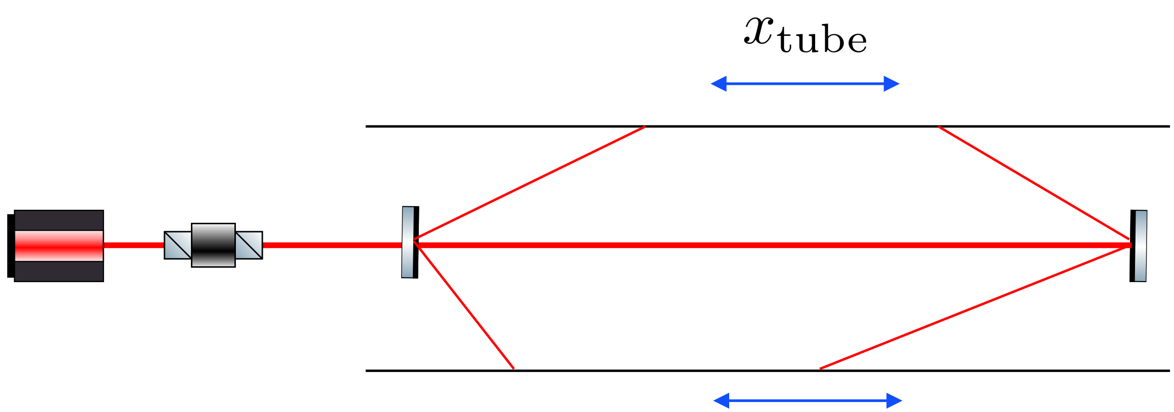




Angular motion is stabilized with bandwidth of 3 Hz. Controls noise is seen up to 30 Hz.

Backscattering

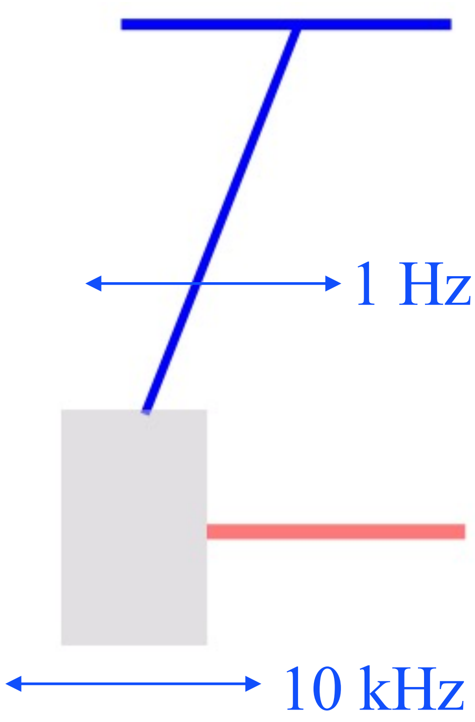
- Small fraction of the beam scatters away from the optic, hits the beam tube, and scatters back into the main beam.



$$h_{\text{noise}} \approx \sqrt{\frac{P_{\text{sc}}}{2P_{\text{arm}}}}\frac{x_{\text{tube}}}{L}$$

Thermal noises

- LIGO detectors operate at room temperature
- Each degree of freedom has energy of kT / 2



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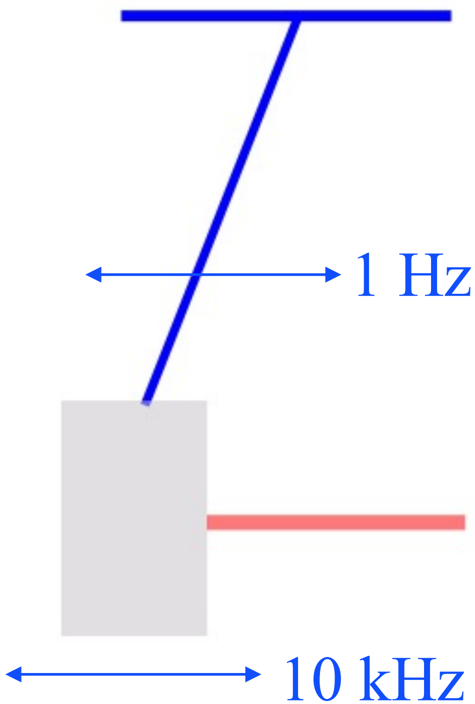

The fluctuation-dissipation theorem:

$$x^2(\Omega) = \frac{4kT}{m\Omega}\left|\mathrm{Im}[\frac{x}{F_{\mathrm{ext}}}]\right|$$

Q is the quality factor of the mechanical mode

$$x^2(\Omega) \underset{\Omega \ll \Omega_0}{\approx} \frac{4kT}{m\Omega\Omega_0^2 Q}$$

$$x^2(\Omega) \underset{\Omega \gg \Omega_0}{\approx} \frac{4kT\Omega_0^2}{m\Omega^5 Q}$$

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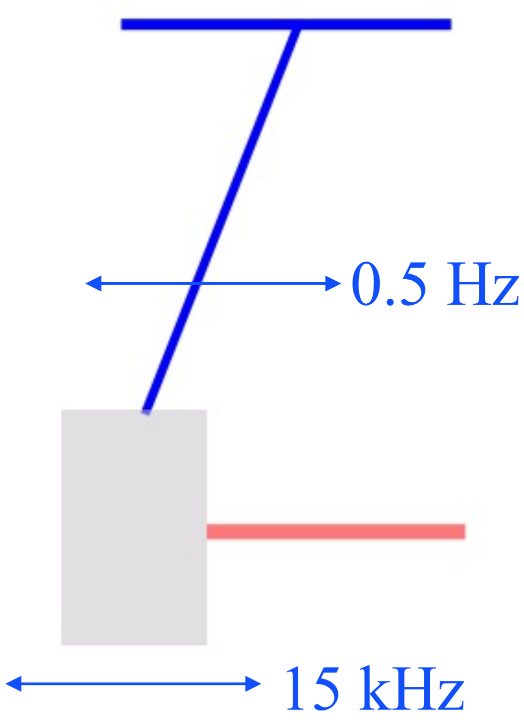
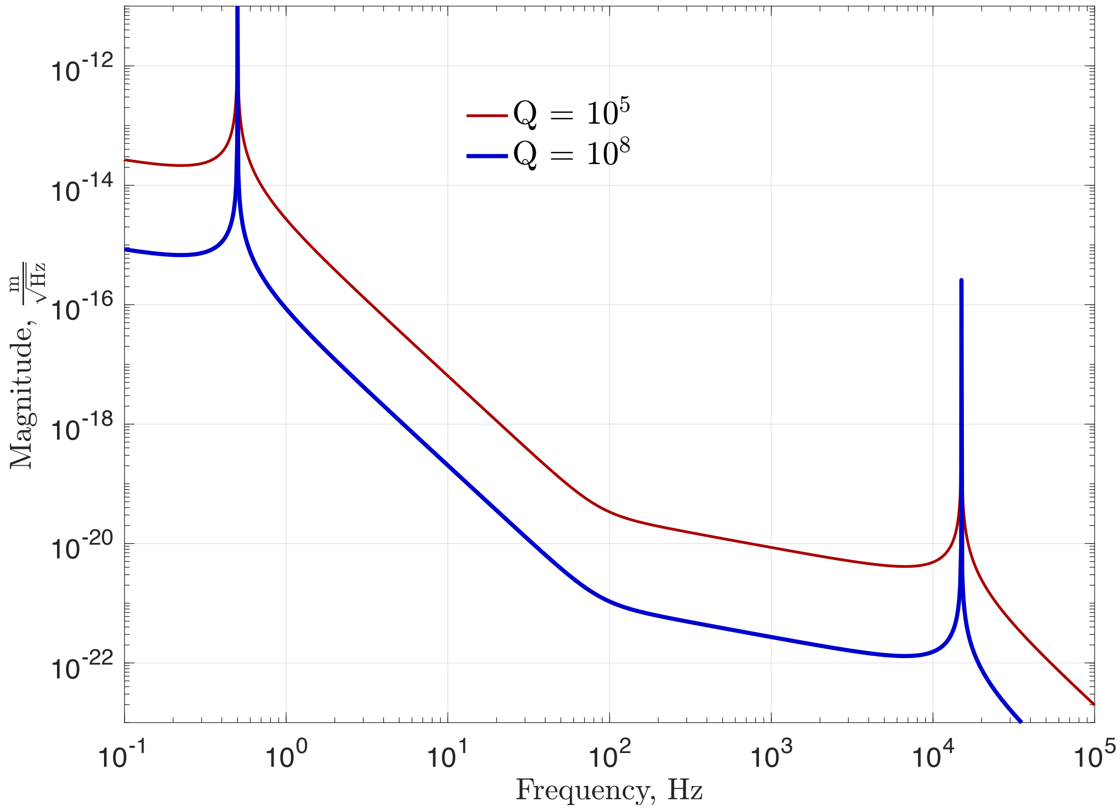




Quantum noise

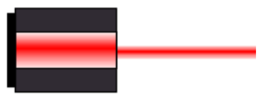
- Laser field consists of a finite number of photons

$$E(t) = E_1(t)\sin(\omega_0 t) + E_2(t)\cos(\omega_0 t)$$

$$\Delta E_1 \Delta E_2 \geq \hbar C$$

- The lower bound is achieved with coherent states

$$|\alpha > = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{n=\infty} \frac{\alpha^n}{\sqrt{n!}} |n >$$

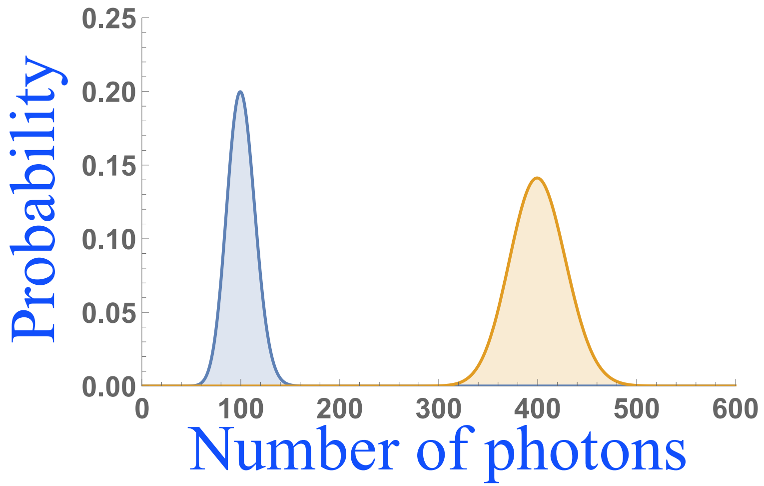
$|0 >$  $|\alpha >$

Power and variance:

$$< \alpha|N|\alpha > = \alpha\alpha^*$$

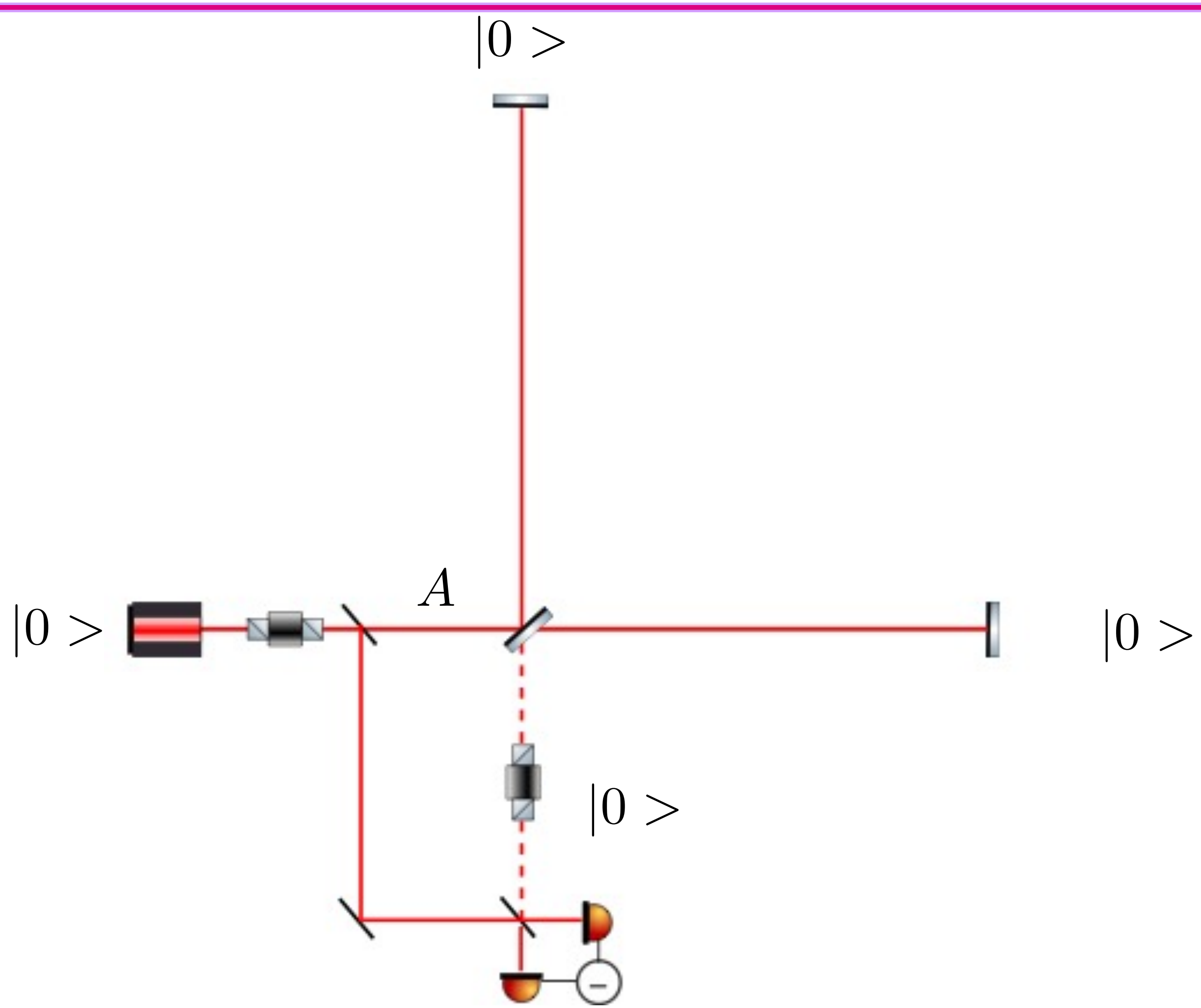
$$< \alpha|N^2|\alpha > - < \alpha|N|\alpha >^2 = \alpha\alpha^*$$

$$\mathrm{SNR} \sim \sqrt{\alpha\alpha^*} \sim \sqrt{P_{\mathrm{laser}}}$$



LIGO

Quantum noise



Quantum noise

- Use Heisenberg picture

$$E(t) = E_1(t)\sin(\omega_0 t) + E_2(t)\cos(\omega_0 t)$$

E_1 is the phase quadrature

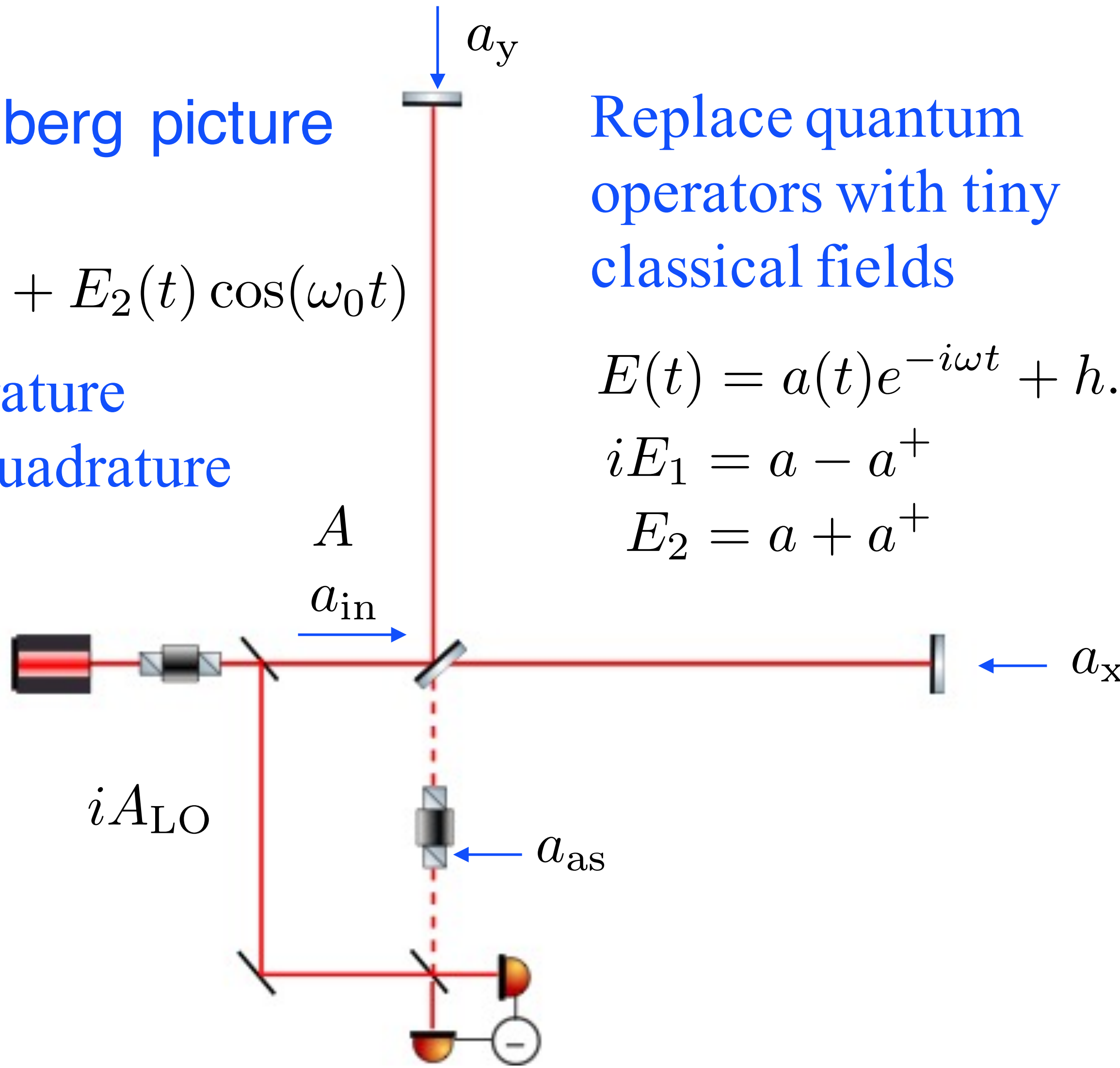
E_2 is the amplitude quadrature

Replace quantum operators with tiny classical fields

$$E(t) = a(t)e^{-i\omega t} + h.c.$$

$$iE_1 = a - a^+$$

$$E_2 = a + a^+$$



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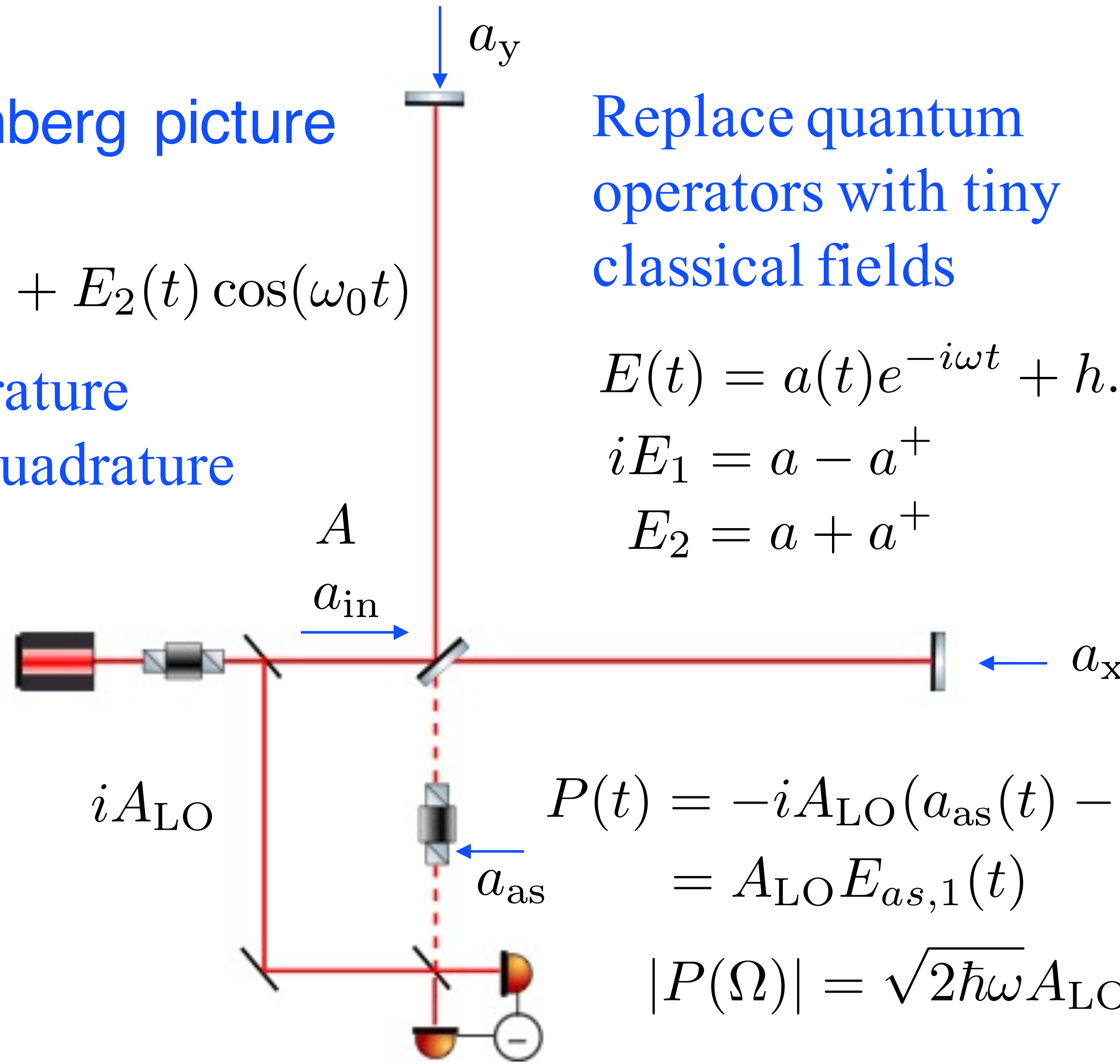


$$P(t) = -iA_{LO}(a_{as}(t) - a_{as}^+(t))$$
$$= A_{LO}E_{as,1}(t)$$

$$|P(\Omega)| = \sqrt{2\hbar\omega}A_{LO}$$

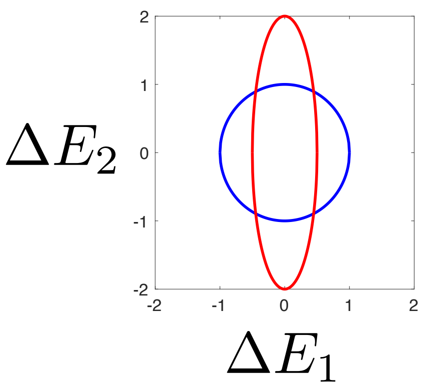
Squeezed states of light

- Keep the uncertainty area the same

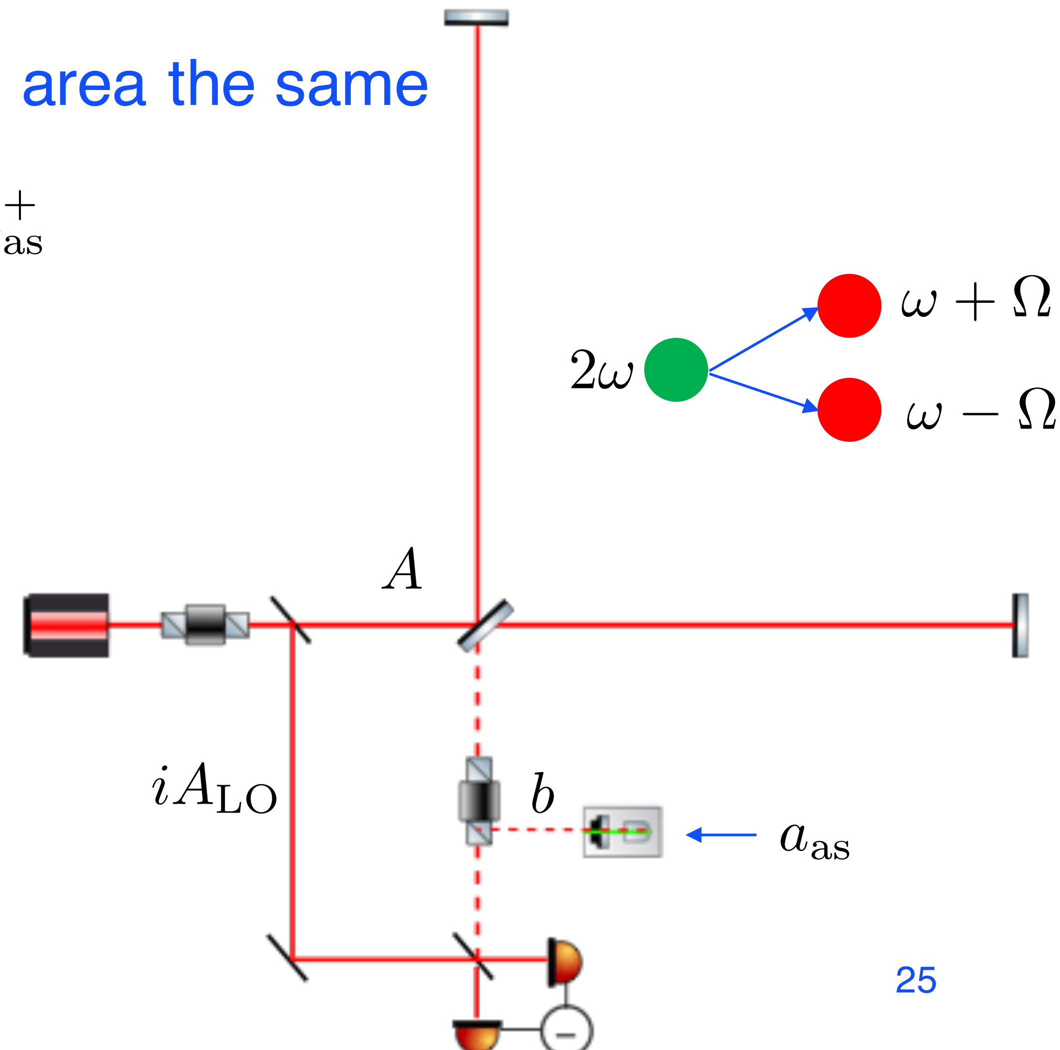
$$b = \cosh(r)a_{\rm as} + \sinh(r)a^+_{\rm as}$$

$$b - b^+ = (a_{\rm as} - a^+_{\rm as})e^{-r}$$

$$|P(\Omega)| = \sqrt{2\hbar\omega}A_{\rm LO}e^{-r}$$



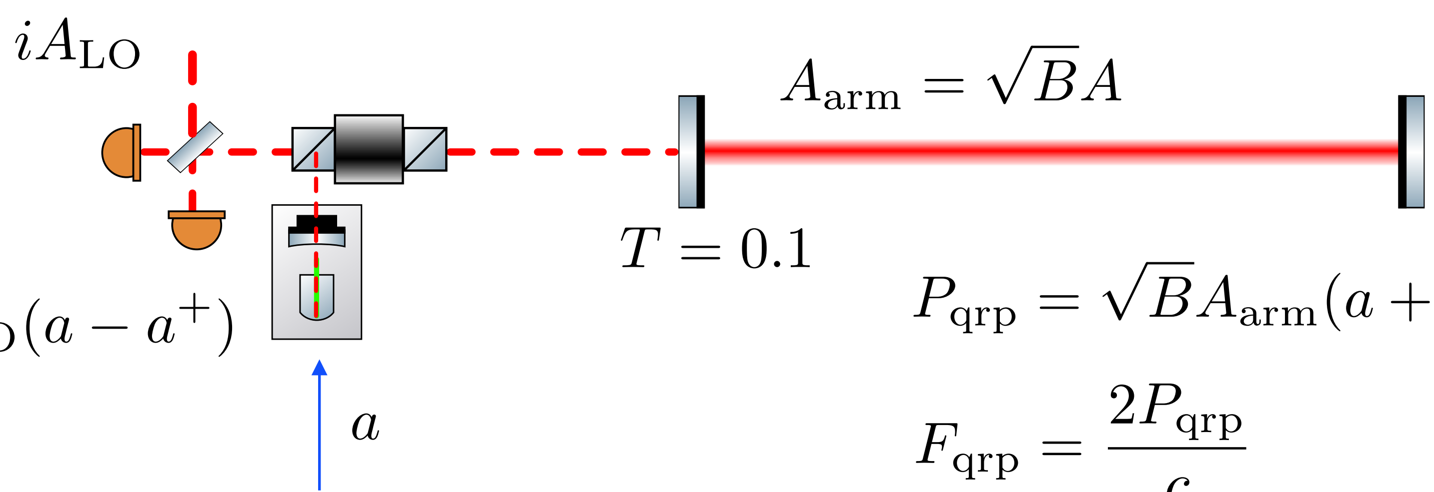
Signal $P = 8\pi A_{\rm LO}A\frac{L}{\lambda}h$



Quantum noise in LIGO

Exercise: calculate quantum noise in the LIGO detectors

The LIGO layout is equivalent to



$$P = iA_{\mathrm{LO}}(a - a^{+})$$

$$P_{\mathrm{qrp}} = \sqrt{B} A_{\mathrm{arm}}(a + a^{+})$$

$$F_{\mathrm{qrp}} = \frac{2P_{\mathrm{qrp}}}{c}$$

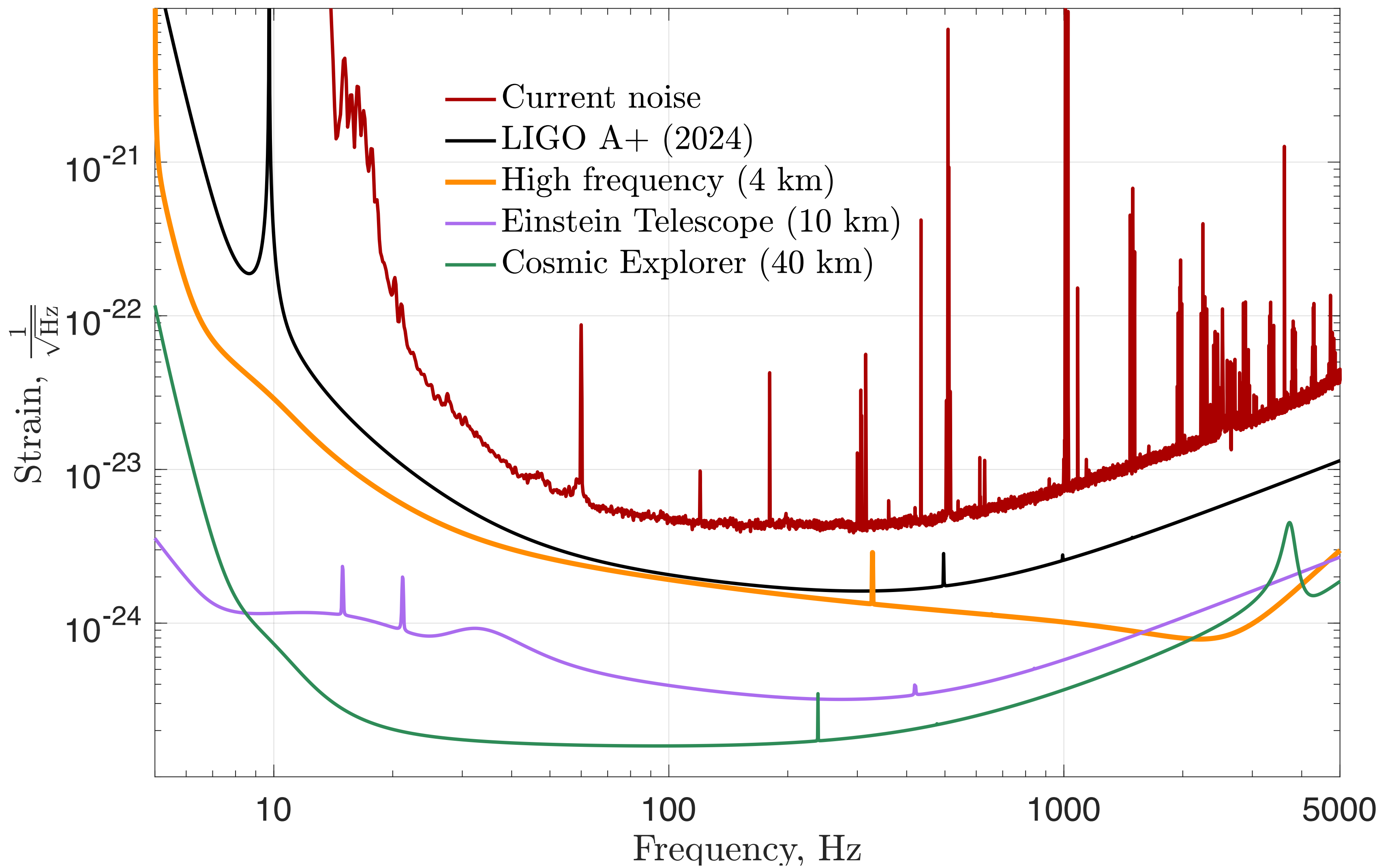
B is the power build-up in the resonator

LIGO

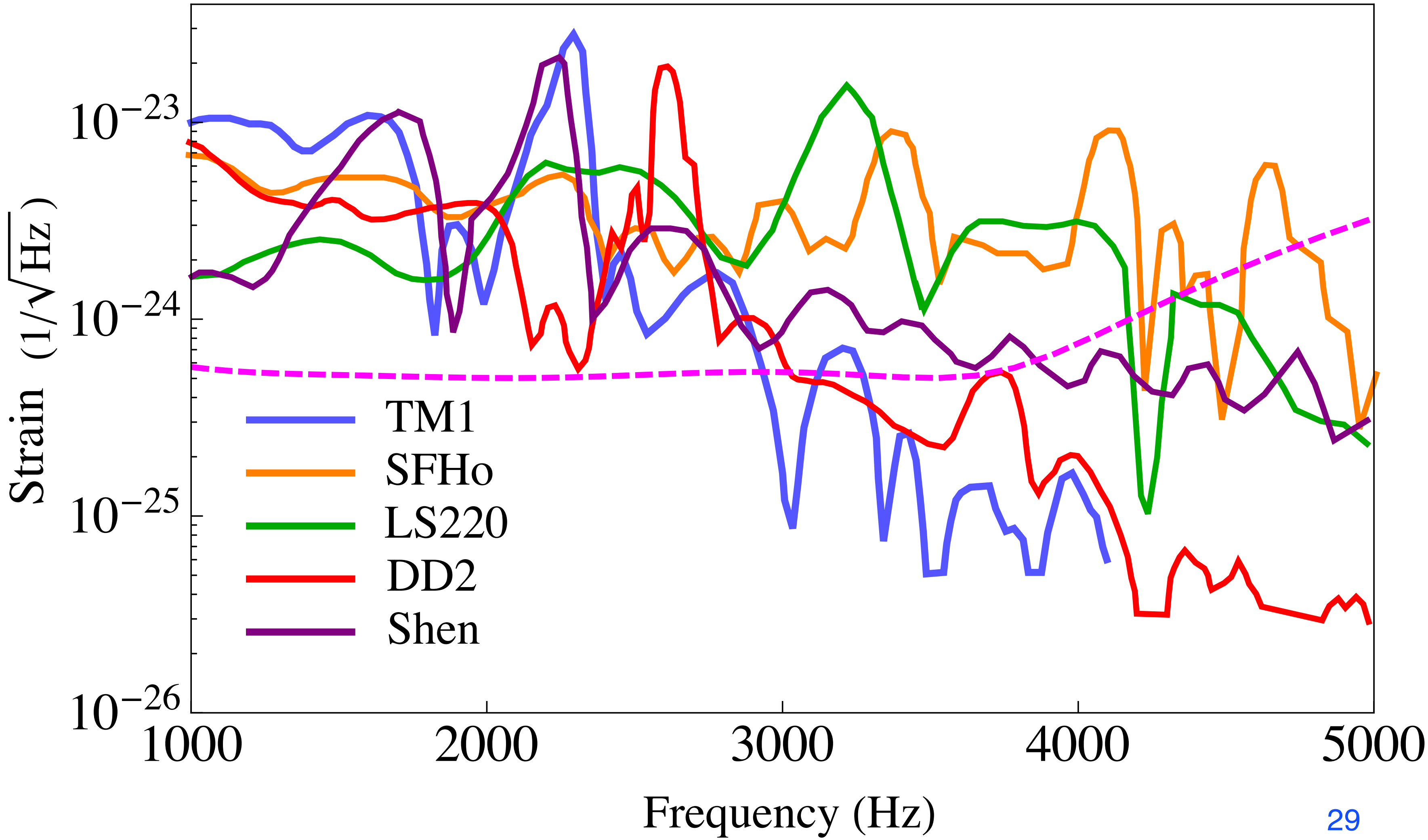
Future directions

- Two more science runs in the current configuration
  - Population of compact binaries
  - More neutron star mergers
  - Neutron star – black hole binary
  - Tests of general relativity
- A+ upgrade (2024): better coatings, improved quantum noise (double the range)
- Possible post A+ upgrade at room temperature (improve high frequencies)
- Cryogenic LIGO, new facilities

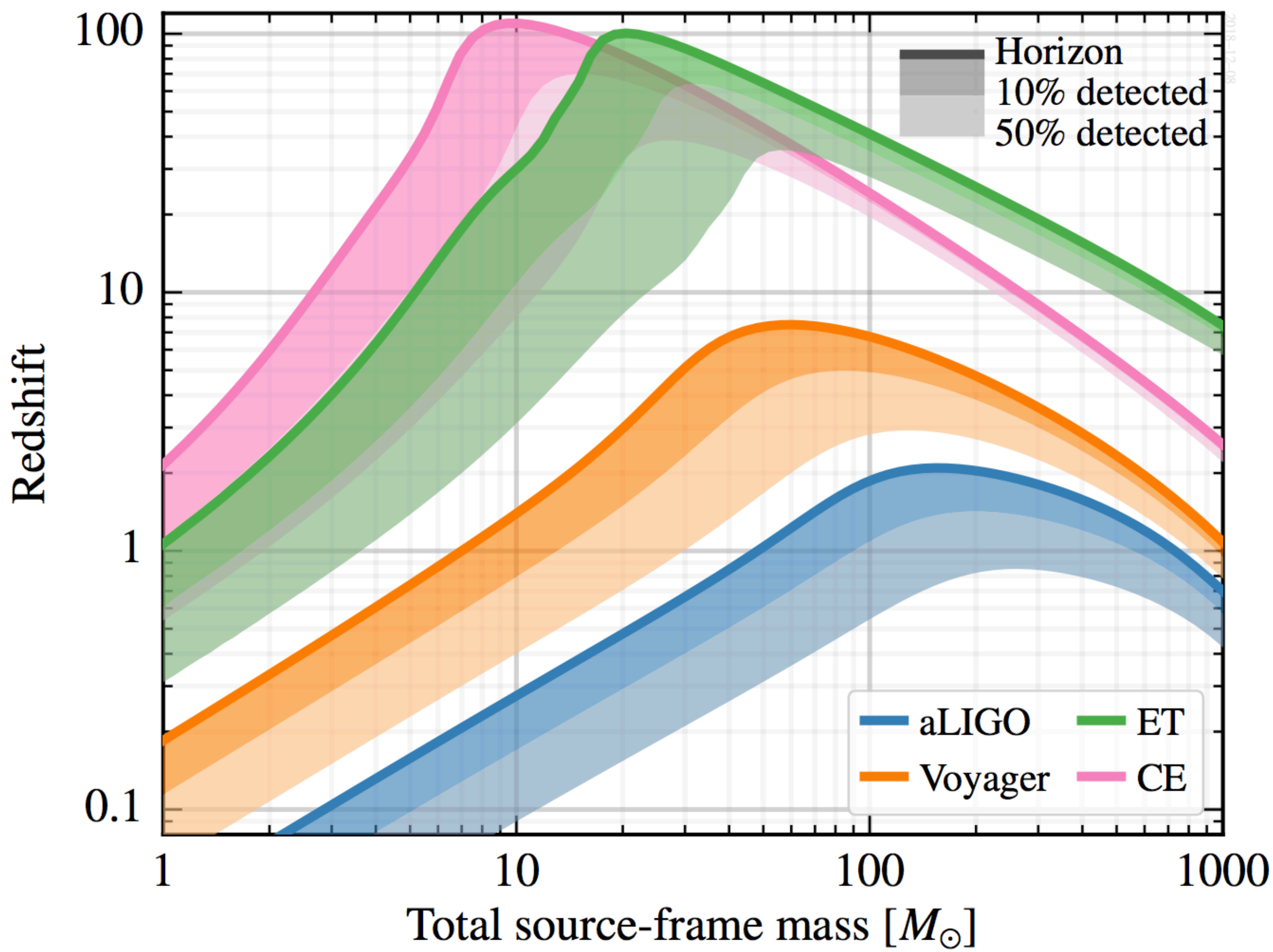
Estimated sensitivities



Neutron star equation of state



Future facilities



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LIGO

Conclusions

- The current LIGO detectors approach their design sensitivity.
- Future studies will target population statistics, physics of neutron stars and cosmology.

Extra slides