Exercises for the multimessenger discussion

1. Derive the expression $\frac{dn}{d\Omega} \propto (1 - \beta \cos\theta_1)$ for the collision rate between a cosmic ray and a cloud moving with the non-relativistic speed V.
   *N*ote: the collision rate is proportional to the relative velocity. Use the fact that the cosmic ray is ultra-relativistic.

2. Find the minimal energy $E_{\rm th}$ of a proton scattering on a photon with the typical energy of the cosmic microwave background ($T \approx 2.7$ K) for the process $p + \gamma \to p + \pi^0$. Guess the cross section of this reaction, check it against the curve at `http://pdg.lbl.gov`, and estimate the mean free path of a proton with $E \gg E_{\rm th}$.
   *N*ote: cross-sections for the resonance processes may be guessed with the use of Breit-Wigner formula.

3. Neutrinos from a type II supernova.
   The proto-neutron formed during the core collapse of a massive star emits copiously neutrinos. Its mass is $\approx 1.4 M_\odot$, and its radius ≈ 15 km. Estimate the total (gravitational potential) energy E_b released. Apply the virial theorem ($-E_{\rm pot} = 2E_{\rm kin}$) to a nucleon N at the surface of the proto-neutron star and estimate its kinetic energy E_N. Estimate the number N_ν of neutrinos emitted and the duration of the neutrino signal (in the random walk picture) using $E_\nu = E_N/2$, $E_b = N_\nu E_\nu$ and $\sigma_\nu = 10^{-43}{\rm cm}^2 (E_\nu/{\rm MeV})^2$. For the case of SN1987A in the Large Magellanic Cloud at a distance of 50 kpc, how many neutrinos were observed (using the same σ_ν) in a detector with 10^{32} protons?